Math 124 K - Autumn 2008
Mid-Term Exam Number Two
November 18, 2008
Answers

1. (a) $$\frac{dy}{dx} = \frac{1}{\cos(\tan x)}(-\sin(\tan x))\sec^2 x$$

   (b) $$\frac{dy}{dx} = 2\sec 2x \tan 2x \tan 3x + 3\sec 2x \sec^2 3x$$

   (c) $$\frac{dy}{dx} = \frac{-\frac{1}{y} - \frac{y}{x}x^y}{x^y \ln x - \frac{x}{y^2}}$$

2. (a) the tangent lines are $x = 0$ and $y = 2(x-1)$ (b) $y \approx 2(1.1-1) = 0.2$.

3. (a) local minimum at $x = 0$, local maximum at $x = 2$ (b) inflection points at $x = 2 \pm \sqrt{2}$ (c) $y = 0$ is a horizontal asymptote for f.

4. (a) 0 (b) 1 (c) -1

5. The area is increasing at the rate of 1.15714 square meters per second.

6. (a) $f'(x) = \dfrac{1}{x} + e^x > 0$ for all $x > 0$, the domain of f. Hence, f is always increasing, and so can have *at most* one root. Also, $f(1) = e > 0$, and $f(0.1) = -1.1197... < 0$, so f has *at least* one root. Thus, f has exactly one root.

   (b) $f''(x) = -\dfrac{1}{x^2} + e^x$. f'' is continuous on $x > 0$, and $f''(1) = e - 1 > 0$ while $f''(0.1) = -98.89.. < 0$, so f'' does change sign, and so f has at least one inflection point.

   (c) With Newton's method, one can find the root is approximately 0.2698741375... .