Math 124 K - Autumn 2009
Mid-Term Exam Number Two
November 24, 2009
Answers/Solutions

1. (a) $\dfrac{dy}{dx} = \sec(x^2+\sin x)\tan(x^2+\sin x)(2x+\cos x)\cos e^x + \sec(x^2+\sin x)(-\sin e^x)(e^x)$

   (b) Using $\ln y = x^x \ln x$ and then $\ln\ln y = x\ln x + \ln\ln x$, you get

   $$\frac{dy}{dx} = x^{x^x} x^x \left(\ln x + (\ln x)^2 + \frac{1}{x}\right)$$

2. With an initial root approximation of $r_0 = 0.5$, you should get

   $$r_1 = 0.49481868, r_2 = 0.49486641, r_3 = 0.49486641$$

   so, to six correct digits, the root is 0.494866.

3. The angle is changing at the rate of -0.00064 radians per second.

4. For the first family, we see that each curve satisfies the relationship

   $$2x + 2ky\frac{dy}{dx} = 0$$

   or

   $$\frac{dy}{dx} = -\frac{x}{ky}.$$

   Solving for k in the equation $x^2 + ky^2 = 1$, we find

   $$k = \frac{1-x^2}{y^2}$$

   and so, for this family,

   $$\frac{dy}{dx} = -\frac{xy}{1-x^2}.$$

   For the other family, we have

   $$2x + 2y\frac{dy}{dx} = \frac{2}{x}$$

   which yields

   $$\frac{dy}{dx} = \frac{1-x^2}{xy}.$$

   Noting that this is the negative reciprocal of the expression for the other family, we are done.

5. The function has two critical points in the interval $[-1, 2]$: 0 and $\left(\dfrac{4}{3}\right)^{3/4} \approx 1.24080647$. Evaluating the function at these points and the endpoints, we find the absolute maximum occurs at $x = \left(\dfrac{4}{3}\right)^{3/4}$ where the function is about 0.7698, and the absolute minimum occurs at $x = 0$, where the function equals 0.

6. (a) There is only one critical point: $x = e^{-1/2}$.
   (b) The function is decreasing on $(0, e^{-1/2})$ and increasing on $(e^{-1/2}, \infty)$.
   (c) There is a local minimum at $x = e^{-1/2}$.
   (d) There is one inflection point at $x = e^{-3/2}$.