Math 124 A, B - Spring 2005
Mid-Term Exam Number Two
Answers

Version A - problem 6 involves the point $(0, \dfrac{1}{3})$.

1. (a) $\dfrac{dy}{dx} = (2x+3)e^{5-3x} + (x^2+3x)e^{5-3x}(-3)$

   (b) $\dfrac{dy}{dx} = \dfrac{(20x^4 - 24x)(6x^2 - 9x) - (4x^5 - 12x^2 + 3)(12x - 9)}{(6x^2 - 9x)^2}$

   (c) $\dfrac{dy}{dx} = \dfrac{1}{2}(\dfrac{1}{2}x + \sin(x^3 - x))^{-1/2}(\dfrac{1}{2} + \cos(x^3 - x)(3x^2 - 1))$

2. (a) $\dfrac{dy}{dx} = \dfrac{x^{\ln x}}{x-1}\left(\dfrac{2\ln x}{x} - \dfrac{1}{x-1}\right)$ (Logarithmic differentiation works well here.)

   (b) $\dfrac{dy}{dx} = -\dfrac{2x^3}{3y^5}$

   (c) $\dfrac{dy}{dx} = \dfrac{-y - \cos x}{x - \cos y}$

3. $y = -4x$.

4. The linear approximation is

$$\ln x \approx \frac{1}{e^2}(x - e^2) + 2$$

   Evaluating at $x = 7$, we can say that

$$\ln 7 \approx \frac{1}{e^2}(7 - e^2) + 2 = 1.9459101....$$

5. (a) The distance to the balloon is changing at -0.83205 meters per second.

   (b) The angle is changing at the rate of 0.09230769 radians per second.

6. Let $(a, a + \frac{1}{a})$ be a point on the curve. If the tangent line at this point passes through $(0, \frac{1}{3})$, then the slope of the tangent line can be expressed as

$$1 - \frac{1}{a^2}$$

   and as

$$\frac{a + \frac{1}{a} - \frac{1}{3}}{a - 0}$$

These must be equal. Setting them equal and solving for a, we find one solution $a = 6$. Hence, the point $(6, 6\frac{1}{6})$ is the only point on the curve that has a tangent line that passes through $(0, \frac{1}{3})$.

Version B - problem 6 involves the point $(0, \frac{2}{5})$

1. (a) $\dfrac{dy}{dx} = \dfrac{(8x^3 - 15x^2)(7x^2 + 3x) - (2x^4 - 5x^3 + 1)(14x + 3)}{(7x^2 + 3x)^2}$

   (b) $\dfrac{dy}{dx} = \dfrac{1}{2}(\dfrac{1}{5}x + \cos(x^4 + 2x))^{-1/2}(\dfrac{1}{5} - \sin(x^4 + 2x)(4x^3 + 2))$

   (c) $\dfrac{dy}{dx} - (3x^2 + 2)e^{6-2x} + (x^3 + 2x)e^{6-2x}(-2)$

2. (a) $\dfrac{dy}{dx} = -\dfrac{4x^7}{3y^5}$

   (b) $\dfrac{dy}{dx} = \dfrac{x^{\ln x}}{x - 5}\left(\dfrac{2\ln x}{x} - \dfrac{1}{x - 5}\right)$ (Logarithmic differentiaion work well here)

   (c) $\dfrac{dy}{dx} = \dfrac{-y - \cos x}{x - \sin y}$

3. $y = \dfrac{2}{7}x$

4. The linear approximation is

$$\ln x \approx 2 + \frac{1}{e^2}(x - e^2)$$

   so

$$\ln 8 \approx 2 + \frac{1}{e^2}(8 - e^2) = 2.0826822....$$

5. (a) The distance to the balloon is changing at -0.83205 meters per second.

   (b) The angle is changing at the rate of 0.09230769 radians per second.

6. Let $(a, a + \frac{1}{a})$ be a point on the curve. If the tangent line at this point passes through $(0, \frac{2}{5})$, then the slope of the tangent line can be expressed as

$$1 - \frac{1}{a^2}$$

   and as

$$\frac{a + \frac{1}{a} - \frac{2}{5}}{a - 0}$$

   These must be equal. Setting them equal and solving for a, we find one solution $a = 5$. Hence, the point $(5, 5\frac{1}{5})$ is the only point on the curve that has a tangent line that passes through $(0, \frac{2}{5})$.