

Math 125 D Autumn 2023
Mid-Term Exam Number Two
November 16, 2023

Solutions

Version A has a 3 somewhere in the integral in problem 1(a).

Version A

1. (a) $\frac{3}{2} \ln|x+3| - \frac{1}{2} \ln|x+1| + C$ (use partial fractions)
(b) $\frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C$
2. (a) $\frac{1}{2}x \sin^2 x - \frac{1}{4}x + \frac{1}{8} \sin 2x + C$ (use integration by parts)
(b) $-16(4-x^2)^{1/2} + \frac{8}{3}(4-x^2)^{3/2} - \frac{1}{5}(4-x^2)^{5/2} + C$ (use trig substitution)
3. (a) $\frac{1}{7}(x^2-1)^{7/2} + \frac{1}{5}(x^2-1)^{5/2} + C$ (use trig substitution)
(b) $\frac{1}{3}x^3 \tan^{-1} x - \frac{1}{6}x^2 + \frac{1}{6} \ln(x^2+1) + C$ (use integration by parts then partial fractions)
4. 14π (the cylindrical shells method works great here)
5. $W = \rho g \int_0^3 \left(\frac{4}{3}\pi y\right) (5-y) dy + \rho g \int_3^5 4\pi(5-y) dy$
6. $\left[\ln \frac{15}{e^3-1}, 3 + \ln \frac{15}{e^3-1} \right]$

Version B

1. (a) $\frac{1}{6} \sin^6 x - \frac{1}{8} \sin^8 x + C$
(b) $2 \ln|x+4| - \ln|x+2| + C$ (use partial fractions)
2. (a) $-4\sqrt{4-x^2} + \frac{1}{3}(4-x^2)^{3/2} + C$ (use trig substitution)
(b) $\frac{1}{3}x \sin^3 x + \frac{1}{3} \cos x - \frac{1}{9} \cos^3 x + C$ (use integration by parts)
3. (a) $\frac{1}{3}x^3 \tan^{-1} x - \frac{1}{6}x^2 + \frac{1}{6} \ln(x^2+1) + C$ (use integration by parts the partial fractions)
(b) $\frac{1}{9}(x^2-1)^{9/2} + \frac{2}{7}(x^2-1)^{7/2} + \frac{1}{5}(x^2-1)^{5/2} + C$ (use trig substitution)
4. 80π (the cylindrical shells method works great here)
5. $W = \rho g \int_0^8 (2\pi y)(9-y) dy + \rho g \int_8^9 16\pi(9-y) dy$
6. $\left[\ln \frac{2}{e-1} 1 + \ln \frac{2}{e-1} \right]$