Math 125U - Winter 2002
Mid-Term Exam
February 6, 2002

Name _____

1	10	
2	10	
3	10	
4	10	
5	10	
6	20	
7	20	
8	10	
9	10	
10	20	
11	10	
Total	140	

- Complete all questions.
- Show all work for full credit.
- You have 120 minutes to complete the exam.

1. Is $\dfrac{x}{x^2+1}$ an antiderivative of $\dfrac{1-x^2}{(x^2+1)^2}$? Explain.

2. Suppose $f''(x) = 4x + 1$, $f'(1) = 2$, and $f(0) = 1$. Find f.

3. The velocity of a rocket is measured every half-second after lift-off. The data is in the following table.

t	0.0	0.5	1.0	1.5	2.0	2.5
$v(t)$ (m/s)	0.0	1.3	2.7	5.3	12.0	22.3

Assuming the velocity was strictly increasing, find best possible lower and upper estimates for the height of the rocket (assuming an initial height of zero) after 2.5 seconds.

4. Suppose

$$\int_2^{10} f(x)\,dx = 12,\ \int_2^{6} f(x)\,dx = -4,\ \int_5^{10} f(x)\,dx = 1.$$

Find $\displaystyle\int_5^{6} f(x)\,dx$.

5. Suppose $g(x) = \displaystyle\int_3^{\ln x} \frac{\ln t}{e^t}\,dt$. Find $g'(x)$.

6. Evaluate the integrals.

(a) $\int e^x \cos(2e^x)\, dx$

(b) $\int (1+t)\sqrt{2+t}\, dt$

7. Evaluate the integrals.

(a) $\int x^9 \sqrt{x^5 - 2}\, dx$

(b) $\int \frac{1}{x\sqrt{\ln x}}\, dx$

8. Find the area of the region bounded by $y = 2x$ and $y = x^2 - 3x$.

9. Find the volume of the solid created by revolving the region bounded by

$$y = e^{x^2},\ x = \sqrt{\ln(\pi + 1)},\ x = 0,\ \text{ and } y = 0$$

about the y-axis.

10. Consider the region bounded by the curve $y = \ln x$ and the line which passes through $(1, 0)$ and $(e, 1)$.

    (a) Set up (but do not evaluate) an integral representing the volume of the solid obtained by revolving this region about the y-axis.

    (b) Set up (but do not evaluate) an integral representing the volume of the solid obtained by revolving this region about the x-axis.

11. Consider the solid created by revolving the region bounded by

$$y = x^3, y = 8 \text{ and the } y - \text{axis}$$

about the y-axis. Suppose a tank with this shape is filled with a heavy liquid weighing 90 lb/ft^3. Calculate the work done in pumping all of the liquid to the top of the tank (assume linear units are feet).