Math 125 C - Winter 2005
Mid-Term Exam Number Two Solutions
February 24, 2005

1. (a) $$-\frac{1}{24}\cos^{24} x + \frac{1}{26}\cos^{26} x + C$$

   (b) A u-substitution and integration by parts get you to
   $$\frac{1}{3}\sin^3 x \ln(\sin x) - \frac{1}{9}\sin^3 x + C$$

2. (a) Trig substitution ($x = 2\sec\theta$) to get
   $$2^5\left(\frac{1}{5}\frac{(x^2-4)^{5/2}}{2^5} + \frac{2}{3}\frac{(x^2-4)^{3/2}}{2^3} + \frac{\sqrt{x^2-4}}{2}\right) + C$$

   (b) Long division, then factor the denominator and use partial fractions to get
   $$\frac{1}{2}x^2 + x + \frac{27}{5}\ln|x-3| + \frac{8}{5}\ln|x+2| + C$$

3. (a) Use $u = x^4$ and then integration by parts to get
   $$\frac{1}{4}x^4 e^{x^4} - \frac{1}{4}e^{x^4} + C$$

   (b) Complete the square and use $x + 3 = \sqrt{11}\tan\theta$ to get
   $$\frac{1}{\sqrt{11}}\tan^{-1}\left(\frac{x+3}{\sqrt{11}}\right) + C$$

4. (a) Use the substitution $u = e^x - 1$ and then partial fractions. The antiderivative is
   $$\ln|e^x - 1| - \ln|e^x| + C$$
   which is divergent as $x \to 0$ so the integral is divergent.

   (b) Using $x = \tan\theta$ gets the antiderivative
   $$-\frac{\sqrt{x^2+1}}{x} + C$$
   The limit is
   $$-1 + \frac{\sqrt{5}}{2}.$$

5. The integral is
   $$\int_0^{32} 900(9.8)\pi y^{2/5}(35-y)\,dy$$
   which evaluates to 41,378,545.1 J.

6. The average value of this function is
   $$\frac{1}{3}k + \frac{1}{2k}$$
   This function of k is minimized when $k = \sqrt{\frac{3}{2}}$.