

Math 125 G - Winter 2009  
Mid-Term Exam Number Two  
February 26, 2009  
Answers

1. (a) partial fractions

$$-\frac{1}{12} \log(|2x+1|) + \frac{1}{3} \log(|x-1|) + \frac{1}{2}x + C$$

- (b) trig substitution

$$-\frac{1}{3}x^2\sqrt{1-x^2} - \frac{2}{3}\sqrt{1-x^2} + C$$

2. substitution (e.g.  $u = \sqrt{x}$ ) then integration by parts

$$2 \left( \cos(\sqrt{x}) (2-x) + 4 \sin(\sqrt{x}) \sqrt{x} \right) + C$$

3. complete the square, then trig substitution

$$\frac{x-2}{96\sqrt{x^2-4x+12}} + \frac{x-2}{24(x^2-4x+12)^{\frac{3}{2}}} + C$$

4. This integral is improper: the integrand is undefined at  $x = 0$ , which is within the limits of integration. Use a substitution (e.g.  $u = e^x$  or  $u = e^x - 1$ ), and then partial fractions to get an antiderivative. Evaluating the appropriate limits, one finds that the integral is divergent.

5. integration by parts

$$\frac{1}{2}$$

6. volume =  $1000\pi g \int_0^3 (4-y)^2(3-y) dy$