Falling Example

You see a rock falling at a height of 100'. One second later its height is 20'. If it was thrown downward from a height of 150', how fast was it thrown?

Let $h(t)$ be the height of the rock t seconds after it is first viewed. So $h(0) = 100$.

A basic newtonian assumption about falling stuff is that

$$h''(t) = -g = -32.2$$

That is, we assume that falling stuff is always accelerating at a constant rate, and that rate is about 32.2 feet per second^2.

Now, we can find $h'(t)$ by "antidifferentiating" $h''(t)$. We know that $h''(t)$ is the derivative of $h'(t)$, so $h'(t)$ is *an* antiderivative of $h''(t)$.

Thinking about it for a moment, we know that

$$\frac{d}{dx}\left(-32.2t\right) = -32.2$$

so it must be the case that

$$h'(t) = -32.2t + C_1$$

where C_1 is some constant. Similarly, we can "antidifferentiate" $h'(t)$ to get $h(t)$:

$$h(t) = -32.2\frac{t^2}{2} + C_1 t + C_2.$$

Now, we really need to know what C_1 and C_2 are in order to answer any questions about this rock. So, we use the information we're given to get equations involving C_1 and C_2 which we can solve.

We know $h(0) = 100$ and $h(1) = 20$, so

$$h(0) = 100 = C2$$

$$h(1) = 20 = -32.2\frac{1}{2} + C_1 + C_2 = -16.1 + C_1 + 100$$
$$C_1 = 20 + 16.1 - 100 = -63.9$$

So:

$$h(t) = -16.1t^2 - 63.9t + 100$$

and the velocity function of the rock is

$$v(t) = h'(t) = -32.2t - 63.9$$

so if we knew *when* the rock was at a height of 150', we could plug that value of t into $v(t)$ to get the velocity.

To find this t, we set $h(t) = 150$ and solve for t. Using the quadrative formula, we find two solutions: $t = -1.072036159$ and -2.8969079. We know the value we want is $t = -1.072036159$ since we were told that the rock was thrown downward.

Pluggin this value of t into $v(t)$ gives the initial velocity of the rock: $v(1.072036159) = -29.380435$ ft/sec.