How to Attack an Integral

We have six basic techniques of integration. Remember that it is possible to evaluate some integrals using more than one method.

1. **Simple Substitution** - Look for an obvious substitution. It is important to be able to recognize functions with their derivatives:

   e.g., $\sin x$ with $\cos x$
   $\tan x$ with $\sec^2 x$
   $\ln x$ with $\frac{1}{x}$

2. **Trigonometric Identities** - If the integrand contains powers of trig functions, then try splitting off a $\cos x$, $\sin x$, $\sec x \tan x$, $\sec^2 x$, $\csc x \cot x$, or $\csc^2 x$. Then look for a substitution. If this doesn't work, then you may want to try using one of the following identities.

$$\cos^2 x + \sin^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$\cot^2 x + 1 = \csc^2 x$$
$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$
$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

   To undo the last identity, it may be helpful to use the identity

$$\sin 2x = 2 \sin x \cos x$$

3. **Trigonometric Substitution** - If the integrand contains any of the expressions $a^2 - x^2$, $a^2 + x^2$, or $x^2 - a^2$, then try the appropriate trig substitution ($x = a \sin\theta$, $x = a \tan\theta$, or $x = a \sec\theta$, respectively).

4. **Completing the Square** - If the integrand contains the square root of a quadratic expression OR if the integrand contains a rational function whose denominator is an irreducible quadratic expression, then you may be able to complete the square and use trig substitution.

5. **Partial Fractions** - If the integrand is a rational function, then factor the denominator into linear and irreducible quadratic factors and find the partial fraction decomposition. Remember to divide first if the degree of the numerator is greater than or equal to the degree of the denominator.

6. **Integration by Parts** - This method works best when the integrand contains a product of functions. Remember that you may have to apply i.b.p. more than once.

Advice from George F. Simmons: "Be observant, thoughtful, flexible and persistent—all of which are of course easier said than done. If a method doesn't work, be ready to try another. Sometimes several methods work. Keep your options open and do things the easy way— if any. And remember that doing a problem more than one way is a good learning experience."

Use this strategy to evaluate the following integrals. You will not be required to turn these in but you may ask questions about them in office hours or on the discussion board.

1. $\int \sin^4 x \cos^3 x \, dx$

2. $\int \frac{dx}{\sqrt{6x - x^2}}$

3. $\int \frac{\cos x \, dx}{\sqrt{1 + \sin x}}$

4. $\int \frac{dx}{\sqrt{(a^2 + x^2)^3}}$

5. $\int \ln \sqrt{1 + x^2} \, dx$

6. $\int \frac{\sin^{-1} x \, dx}{\sqrt{1 - x^2}}$

7. $\int \frac{(3x - 7) \, dx}{(x-1)(x-2)(x-3)}$

8. $\int e^{\ln \sqrt{x}} \, dx$

9. $\int x^2 e^x \, dx$

10. $\int \frac{\tan x \, dx}{\cos^2 x}$

11. $\int \frac{\cos \sqrt{x} \, dx}{\sqrt{x}}$

12. $\int \frac{(x+1) \, dx}{x^2 (x-1)}$

13. $\int \tan^{-1} 10x \, dx$

14. $\int \frac{dx}{(x^2 - 1)^{3/2}}$

15. $\int x^{2/3} \left(x^{5/3} + 1\right)^{2/3} \, dx$

16. $\int \tan^3 3x \sec 3x \, dx$

17. $\int e^x \cos 2x \, dx$

18. $\int \frac{4x + 1}{x^3 + 4x} \, dx$

19. $\int \frac{dx}{x^2 + 4x + 8}$

20. $\int x^2 \sin x \, dx$

21. $\int \sin^4 x \cos^2 x \, dx$

22. $\int \frac{x^5 \, dx}{\sqrt{1 + x^2}}$

23. $\int \frac{\cos x \, dx}{\sin^3 x - \sin x}$

24. $\int x \tan^2 x \, dx$

25. $\int \tan^2 x \sec^2 x \, dx$

26. $\int x \ln \sqrt{x + 2} \, dx$

27. $\int \frac{x \, dx}{x^4 - 16}$

28. $\int \frac{x^3 + x^2}{x^2 + x - 2} \, dx$

29. $\int \frac{dx}{x^{1/5}}$

30. $\int \sin x \cos 2x \, dx$

31. $\int \frac{dx}{x^4 + 4x^2 + 3}$

32. $\int \frac{x^3 \, dx}{\sqrt{16 - x^2}}$

33. $\int \sin^3 2x \cos^3 2x \, dx$

34. $\int \frac{dx}{x (2 + \ln x)}$

35. $\int \frac{\cot x \, dx}{\ln (e \sin x)}$

36. $\int \frac{\sec^2 x \, dx}{\sec^2 x - 3 \tan x - 1}$