Summary for Midterm Two - Math 125

Here is an outline of the material for the second midterm. The core of your studying should be the assigned homework problems: make sure you really understand those well before moving on to other things (like old midterms).

- 6.4 - Work
  - Many problems finding the work required to perform a certain task can be solved by cutting the task into pieces, approximating the work to perform each piece, summing these approximations, and taking a limit as the number of pieces goes to infinity. The result is, of course, an integral.
  - There are basically three categories of problems you might see in this course: cable problems, spring problems, and pumping/digging problems. You should know how to deal with all of them.
- 6.5 - Average Value of a Function
  - This is a very short section. You should understand the definition of the average value of a function on an interval.
- 7.1 - Integration by Parts
  - You should understand how to apply the *integration by parts* technique:

$$\int u\,dv = uv - v\int du$$

  - You should be able to recognize integrands for which this technique is particularly appropriate, such as:
    1. a positive integer power of x times $\sin x$, $\cos x$, e^x (or related functions)
    2. $\ln x$ times any power of x
    3. e^x times $\sin x$ or $\cos x$
  - Integration by parts can be the method of last resort: if nothing else works, you can almost always try "parts": if you can differentiate the integrand, you can use it. Whether it helps or not is another question, but it works well with such function as $\ln x$ and $\arcsin x$.
- 7.2 - Trigonometric Integrals
  - You should be able to integrate integrals of the form

$$\int \sin^n x \cos^m x\,dx$$

and

$$\int \tan^n x \sec^m x\,dx$$

- The strategies for these integrals depend on the *parities* of m and n (that is, whether they are even or odd). You should practice all cases. Note that the case where the power of $\tan x$ is even and the power of $\sec x$ is odd does not have a clear strategy, so a variety of techniques may be needed.

- 7.3 - Trigonometric Substitution
  - This technique exploits trigonometric Pythagorean identities to give useful substitutions that convert quadratic expressions into squares of trig functions. This makes it particularly useful for eliminating square roots (i.e., when you see a quadratic expression inside a square root, there's a good chance that a trig substitution might be useful).
  - One thing that makes this technique more work than a "simple" substitution is the work required to convert back to the original variable once an antiderivative is found. One efficient way to do this is the "triangle method" as described in lecture and the text. Be sure to practice this aspect of this technique.
- 7.4 - Partial Fractions
  - This technique is applicable to integrands that are rational functions.
  - You should know how to apply this technique to any rational function with a denominator that is factorable as a product of linear and distinct quadratic factors
  - The idea behind this method is completely algebraic: a rational function whose denominator is a product of linear factors can be expressed as the sum of simpler rational functions, each of which has a denominator which is linear or quadratic, and a constant or linear numerator. Such simpler functions are easily integrated.
  - **Long division of polynomials** may be a necessary first step when the numerator has degree equal to or greater than the degree of the denominator.
- 7.5 - Strategies for Integration
  - This section is perhaps the most important. On the exam, the integrals will simply be presented to you for you to solve. You will have to decide which technique, or techniques, to apply to find the antiderivative.
  - There are 81 problems in this section, and they are pretty much all good practice.
- 7.7 - Numerical Integration
  - When we want to evaluate a definite integral, and we cannot find an antiderivative of the integrand, we can approximate the value of the integral. Three techniques we've seen for doing this are
    * Midpoint Rule

    * Trapezoid Rule
    * Simpson's Rule

  All three methods are quite similar, and require us to "sample" the function at a number of equally spaced values of x, then combine these value according to a certain formula which depends on the method.

  Note the two distinct uses of these methods: applied to an integral of an explicitly given function for which we cannot find an antiderivative, and to an integral of a function given only through a table of values (such as example 5 in the text). The implementation of the method is identical in both cases.

- 7.8 - Improper Integrals

  - There are two types of improper integrals:
    1. Those of the form
    $$\int_a^\infty f(x)\,dx, \int_{-\infty}^a f(x)\,dx, \text{or} \int_{-\infty}^\infty f(x)\,dx$$
    2. Those of the form
    $$\int_a^b f(x)\,dx$$
    where $f(x)$ is discontinuous somewhere on the interval $[a, b]$.
  - We treat improper integrals by defining them as the limits of "proper" integrals. For instance,
  $$\int_a^\infty f(x)\,dx = \lim_{k\to\infty} \int_a^k f(x)\,dx$$
  - Be sure you remember l'Hospital's rule, since it is often the tool needed to evaluate the limits resulting from improper integrals.

- 8.1 - Arc Length

  - We can express the arc length S of a graph $y = f(x)$ over the interval $a \le x \le b$ via an integral:
  $$S = \int_a^b \sqrt{1 + (f'(x))^2}\,dx$$
  - For the vast majority of functions f we encounter, the above integral will not be on we can evaluate exactly, since we will be unable to find the antiderivative of the integrand. So, arc length problems on exams tend to be of two types: either they involve a cleverly chosen function for which the integrand is anti-differentiable, or you will be asked to approximate the arc length (using, e.g., Simpson's rule).