

Math 126G - Spring 2002
First Mid-Term Exam
April 23, 2002

Name _____

Section _____

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total	70	

- Complete all questions.
- You may use a calculator during this examination. Other calculating devices are not allowed.
- You may use one hand-written 8.5 by 11 inch page of notes.
- Show all work for full credit.
- You have 50 minutes to complete the exam.

1. Indicate whether each of the following statements is true or false by circling T or F.

T F If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \rightarrow \infty} a_n = 0$.

T F If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ must converge.

T F If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=100}^{\infty} a_n$ must converge.

T F If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverge, then $\sum_{n=1}^{\infty} (a_n + b_n)$ must diverge.

T F If $\lim_{n \rightarrow \infty} a_n = L$, then $\sum_{n=1}^{\infty} a_n = L$.

2. Determine whether each sequence converges or diverges. If it converges, find the limit.

(a) $\left\{(-1)^n \sin\left(\frac{2}{n}\right)\right\}$

(b) $\left\{\left(n + \frac{1}{n}\right)^2 - n^2\right\}$

3. Determine whether the series $\sum_{n=2}^{\infty} (-1)^n \frac{3}{4^n}$ converges or diverges. If it converges, find its sum.

4. By comparing it with an integral, give an upper bound for the series $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$. That is, find a value A so that

$$\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2} < A.$$

5. Determine whether each of the following series converge or diverge. Explain your answer and show all work.

(a)
$$\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$$

(b)
$$\sum_{n=0}^{\infty} \frac{2^n}{(n!)^2}$$

6. Consider the power series

$$\sum_{n=0}^{\infty} (-2)^n \frac{x^n}{n+1}.$$

(a) Find all values of x for which the series converges.

(b) What is the radius of convergence of this series?

7. Determine a power series for the function $f(x) = \frac{x^2}{1 - 2x}$.