Math 126 E, F Spring 2019
Mid-Term Exam Number One
May 2, 2019
Answers

There were two versions in use.
Version A: The first plane given in problem one was $x + y + z = 5$.

1. The plane is $16x + 4y + 14z = 50$.
2. The closest point is $\left(\frac{26}{7}, -\frac{3}{7}, \frac{6}{7}\right)$.
3. The angle between the curves at their point of intersection is $54.7356...^\circ$.
4. (a) The line is $x = -1 + t, y = 80 + 56t, z = 60 + 47t$. (b) With $\vec{r}'(t) = \langle 1, 3t^2 + 2t, 3t^2 - 1\rangle$ and $\vec{r}''(t) = \langle 0, 6t + 2, 6t\rangle$, we can tell that $\vec{r}'$ and $\vec{r}''$ are never parallel, since there is no scalar k that we could multiply $\vec{r}''$ by to get $\vec{r}'$, due to the first component of $\vec{r}''$ being zero and the first component of $\vec{r}'$ not being zero. Also, we can note that $\vec{r}''$ is never the zero vector. Hence, the direction of motion is always changing.
5. (a)The curvature is
$$\kappa = \frac{8}{26^{3/2}}.$$
(b) The radius of curvature is $\frac{1}{8}((2t-1)^2 + (2t+3)^2)^{3/2}$ which is minimized where
$$(2t-1)^2 + (2t+3)^2$$
is minimized. Since this is a quadratic function with positive leading coefficient, the minimum occurs at the vertex: $t = -\frac{1}{2}$.

Version B: The first plane given in problem one was $3x + y + 2z = 3$.

1. The plane is $25x - y + 19z = 39$.
2. The closest point is $\left(-\frac{8}{7}, -\frac{9}{7}, \frac{24}{7}\right)$.
3. The angle between the curves at their point of intersection is $65.905157...^\circ$.
4. (a) The line is $x = 2+4t, y = 2+t, z = t$. (b) With $\vec{r}'(t) = \langle 3t^2+1, 1, 3t^2-2t\rangle$ and $\vec{r}''(t) = \langle 6t, 0, 6t-2\rangle$, we can tell that $\vec{r}'$ and $\vec{r}''$ are never parallel, since there is no scalar k that we could multiply $\vec{r}''$ by to get $\vec{r}'$, due to the second component of $\vec{r}''$ being zero and the second component of $\vec{r}'$ not begin zero. Also, we can note that $\vec{r}''$ is never the zero vector. Hence, the direction of motion is always changing.
5. (a) The curvature is
$$\kappa = \frac{10}{53^{3/2}}.$$
(b) The radius of curvature is $\frac{1}{10}((2t+1)^2 + (2t-4)^2)^{3/2}$ which is minimized where
$$(2t+1)^2 + (2t-4)^2$$
is minimized. Since this is a quadratic function with positive leading coefficient, the minimum occurs at the vertex: $t = \frac{3}{4}$.