Math 300 D - Autumn 2014
Final Exam
December 9, 2014

Name: ____________________ Student ID no. : ____________________

Signature: ____

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

- Check that your exam has six questions.
- Complete all six questions.
- You have 110 minutes to complete the exam.

1. Let $\mathcal{F}$ and $\mathcal{G}$ be families of sets. Prove that $(\cap\mathcal{F}) \cap (\cap\mathcal{G}) = \cap(\mathcal{F} \cup \mathcal{G})$.

2. Let a and b be integers. Prove that $a(b + a + 1)$ is odd iff a and b are both odd.

3. Let A, B, C, and D be sets. Suppose $A \cap C = B \cap D = \varnothing$. Suppose $A \sim B$ and $C \sim D$. Prove that $A \cup C \sim B \cup D$.

4. (a) Give an example of a function $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ such that f is one-to-one, but not onto (i.e., f is injective but not surjective). Prove that f is one-to-one and not onto.

(b) Give an example of a function $g : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ such that g is onto, but not one-to-one (i.e., g is surjective, but not injective). Prove that g is onto and not one-to-one.

5. Use induction to prove that $n! > n^2$ for all integers $n \geq 4$.

6. Let R be the relation defined on the real numbers, $\mathbb{R}$, by

$$(x, y) \in R \Leftrightarrow \text{there exist positive integers } n \text{ and } m \text{ such that } x^n = y^m.$$

Prove that R is an equivalence relation.

DeMorgan's laws

$$\neg(P \wedge Q) \text{ is equivalent to } \neg P \vee \neg Q$$

$$\neg(P \vee Q) \text{ is equivalent to } \neg P \wedge \neg Q$$

Commutative Laws

$$P \wedge Q \text{ is equivalent to } Q \wedge P$$

$$P \vee Q \text{ is equivalent to } Q \vee P$$

Associative Laws

$$P \wedge (Q \wedge R) \text{ is equivalent to } (P \wedge Q) \wedge R$$

$$P \vee (Q \vee R) \text{ is equivalent to } (P \vee Q) \vee R$$

Idempotent Laws

$$P \wedge P \text{ is equivalent to } P$$

$$P \vee P \text{ is equivalent to } P$$

Distributive Laws

$$P \wedge (Q \vee R) \text{ is equivalent to } (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \text{ is equivalent to } (P \vee Q) \wedge (P \vee R)$$

Absorption Laws

$$P \vee (P \wedge Q) \text{ is equivalent to } P$$

$$P \wedge (P \vee Q) \text{ is equivalent to } P$$

Double Negation Law

$$\neg\neg P \text{ is equivalent to } P$$

Tautology Laws

$P \wedge$ (a tautology) is equivalent to P

$P \vee$ (a tautology) is a tautology

$\neg$(a tautology) is a contradiction

Contradiction Laws

$P \wedge$ (a contradiction) is a contradiction

$P \vee$ (a contradiction) is equivalent to P

$\neg$(a contradiction) is a tautology

Conditional Laws

$$P \to Q \text{ is equivalent to } \neg P \vee Q$$

$$P \to Q \text{ is equivalent to } \neg(P \wedge \neg Q)$$

Contrapositive Laws

$$P \to Q \text{ is equivalent to } \neg Q \to \neg P$$

Quantifier Negation Laws

$$\neg\exists x P(x) \text{ is equivalent to } \forall x \neg P(x)$$

$$\neg\forall x P(x) \text{ is equivalent to } \exists x \neg P(x)$$

Sets

$$A = B \Leftrightarrow ((x \in A) \Leftrightarrow (x \in B))$$
$$x \in A \cup B \Leftrightarrow ((x \in A) \vee (x \in B))$$
$$x \in A \cap B \Leftrightarrow ((x \in A) \wedge (x \in B))$$
$$x \in A \setminus B \Leftrightarrow (x \in A) \wedge (x \notin B)$$