Math 300 A - Spring 2012
Final Exam
June 4, 2012

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

1	8	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	58	

- Complete all six questions.
- You have 110 minutes to complete the exam.

.

1. Assign "true" or "false" to each of the following statements. No justification need be given.

   (a) If there is a function $f : \mathbb{Z} \to A$, then A is countable.

   (b) The function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x^3$ is a bijection.

   (c) If A and B are sets, then B and $A \setminus B$ are disjoint.

   (d) If A is a set, and $D \subseteq A \times A$, then D is a relation.

   (e) If $f : \mathbb{Z} \to \mathbb{Z}$, and f is onto, then f is one-to-one.

   (f) There exist one-to-one functions from $\mathbb{R}$ to $\mathbb{Z}$.

   (g) The set $\{-5, 3, 4\}$ is an element of $\mathcal{P}(\mathbb{Z})$.

   (h) Every subset of $\mathbb{Q}$ is countable.

2. Let $A = \mathcal{P}(\mathbb{R})$. Define $f : \mathbb{R} \to A$ by the formula

$$f(x) = \{y \in \mathbb{R} : y^2 < x\}.$$

(a) Is f one-to-one? Prove your answer.

(b) Is f onto? Prove your answer.

3. Let R be a relation on $\mathbb{Q}$ defined by $(p/q, r/s) \in R \Leftrightarrow ps = qr$. Show that R is an equivalence relation.

4. Give a proof by induction that 6 divides $n^3 - n$ for all $n \in \mathbb{Z}_{\geq 0}$.

5. Suppose $f : A \to C$ and $g : B \to C$. Prove that if A and B are disjoint, then

$$f \cup g : A \cup B \to C.$$

6. Suppose R and S are equivalence relations on a set A and $A/R = A/S$. Prove that $R = S$.