Math 300 B - Spring 2012
Final Exam
June 6, 2012

Name: _____ Student ID no. : _____

Signature: _____

1	8	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	58	

- Complete all six questions.
- You have 110 minutes to complete the exam.

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1. Assign "true" or "false" to each of the following statements. No justification need be given.

   (a) If A and B are sets, then $B \setminus A$ and $A \setminus B$ are disjoint.

   (b) There exist one-to-one functions from $\mathbb{R}$ to $\mathbb{Z}$.

   (c) If $f : \mathbb{Z} \to \mathbb{Z}$, and f is onto, then f is one-to-one.

   (d) The set $\{1, 2, 4, 5\}$ is an element of $\mathcal{P}(\{1, 2, 3, 4, 5\})$.

   (e) If A is a set, and $D \subseteq A \times A$, then D is a relation.

   (f) If there is a function $f : \mathbb{Z} \to A$, then A is countable.

   (g) The function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = (x + 4)^3$ is a bijection.

   (h) There exist uncountable subsets of $\mathbb{Z} \times \mathbb{Z}$.

2. Suppose $f : A \to B$ and f is one-to-one. Prove that there is some set $B' \subseteq B$ such that $f^{-1} : B' \to A$.

3. Let $A = \mathcal{P}(\mathbb{R})$. Define $f : \mathbb{R} \to A$ by the formula

$$f(x) = \{y \in \mathbb{R} : y^2 < x\}.$$

(a) Is f one-to-one? Prove your answer.

(b) Is f onto? Prove your answer.

4. Let $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x - y \in \mathbb{Z}\}$.

   Is S an equivalence relation? Prove your answer.

5. Use induction to prove that 49 divides $36^n + 14n - 1$ for all $n \in \mathbb{Z}_{\geq 0}$.

6. Suppose R is an equivalence relation on a set A.

   Prove that for every $x \in A$ and $y \in A$, $y \in [x]_R$ iff $[y]_R = [x]_R$.