Math 300 D - Winter 2014
Final Exam
March 18, 2014

Name: ______________________________ Student ID no. : ____________________

Signature: ____

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

- Check that your exam has six questions.
- Complete all six questions.
- You have 110 minutes to complete the exam.

1. Let A and B be disjoint sets. Suppose there is a bijection from A to I_n and there is a bijection from B to I_m. Prove that there exists a bijection from $A \cup B$ to I_{m+n}.

2. Use induction to prove that, for all integers $n \geq 0$,

$$8 | 5^n + 12n - 1.$$

3. (a) Let A be the set of all real functions $f : \mathbb{R} \to \mathbb{R}$. Define a relation R on A by:

$$(f, g) \in R \Leftrightarrow \text{there exists a real constant } k \text{ such that } f(x) = g(x) + k \text{ for all } x \in \mathbb{R}.$$

Prove that R is an equivalence relation.

(b) Define a relation R on $\mathbb{R}$ by:

$$(x, y) \in R \Leftrightarrow |x - y| < 1$$

Prove that R is not an equivalence relation.

4. Let A and B be sets. Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

5. (a) Let $m \in \mathbb{Z}$ and suppose $m > 1$. Suppose $a, b, c \in \mathbb{Z}$.
Prove that if $a \equiv b \pmod{m}$, then $ac \equiv bc \pmod{m}$.

(b) Prove that if n is an integer then $n^2 \equiv 0, 1,$ or $4 \pmod{8}$.

6. Let A, B and C be sets. Let $f : A \to B$, and $g : B \to C$.

   (a) Suppose $g \circ f : A \to C$ is one-to-one. Is f necessarily one-to-one? Prove your answer.

   (b) Suppose $g \circ f : A \to C$ is one-to-one. Is g necessarily one-to-one? Prove your answer.

DeMorgan's laws

$$\neg(P \wedge Q) \text{ is equivalent to } \neg P \vee \neg Q$$

$$\neg(P \vee Q) \text{ is equivalent to } \neg P \wedge \neg Q$$

Commutative Laws

$$P \wedge Q \text{ is equivalent to } Q \wedge P$$

$$P \vee Q \text{ is equivalent to } Q \vee P$$

Associative Laws

$$P \wedge (Q \wedge R) \text{ is equivalent to } (P \wedge Q) \wedge R$$

$$P \vee (Q \vee R) \text{ is equivalent to } (P \vee Q) \vee R$$

Idempotent Laws

$$P \wedge P \text{ is equivalent to } P$$

$$P \vee P \text{ is equivalent to } P$$

Distributive Laws

$$P \wedge (Q \vee R) \text{ is equivalent to } (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \text{ is equivalent to } (P \vee Q) \wedge (P \vee R)$$

Absorption Laws

$$P \vee (P \wedge Q) \text{ is equivalent to } P$$

$$P \wedge (P \vee Q) \text{ is equivalent to } P$$

Double Negation Law

$$\neg\neg P \text{ is equivalent to } P$$

Tautology Laws

$P \wedge$ (a tautology) is equivalent to P

$P \vee$ (a tautology) is a tautology

$\neg$(a tautology) is a contradiction

Contradiction Laws

$P \wedge$ (a contradiction) is a contradiction

$P \vee$ (a contradiction) is equivalent to P

$\neg$(a contradiction) is a tautology

Conditional Laws

$$P \rightarrow Q \text{ is equivalent to } \neg P \vee Q$$

$$P \rightarrow Q \text{ is equivalent to } \neg(P \wedge \neg Q)$$

Contrapositive Laws

$$P \rightarrow Q \text{ is equivalent to } \neg Q \rightarrow \neg P$$

Quantifier Negation Laws

$$\neg \exists x P(x) \text{ is equivalent to } \forall x \neg P(x)$$

$$\neg \forall x P(x) \text{ is equivalent to } \exists x \neg P(x)$$

Sets

$$A = B \Leftrightarrow ((x \in A) \Leftrightarrow (x \in B))$$

$$x \in A \cup B \Leftrightarrow ((x \in A) \vee (x \in B))$$

$$x \in A \cap B \Leftrightarrow ((x \in A) \wedge (x \in B))$$

$$x \in A \setminus B \Leftrightarrow (x \in A) \wedge (x \notin B)$$