Math 300 D - Autumn 2014
Midterm Exam Number One
October 15, 2014

Name: ____________________ Student ID no. : ____________________

Signature: ____

1	10	
2	10	
3	10	
4	10	
5	9	
Total	49	

- Complete all five questions.
- You have 50 minutes to complete the exam.

1. Use a truth table to decide whether the following argument is valid. Explain your conclusion.

   - Angela will be there, and Boris or Chen will be there.
   - Angela will not be there, or Chen will not be there.
   - Therefore, Boris and Chen will be there.

2. Let A,B,and C be sets. Verify the following identities by showing that that the statement $x \in$ "the left hand set" is equivalent to the statement $x \in$ "the right hand set". Give a justification for each step (e.g., definition of union, DeMorgan's law, distributive law, etc.)

   (a) $(A \cap (B \cup C)) \cap C = A \cap C$

   (b) $(A \cup B) \setminus (A \cup C) = B \setminus (A \cup C)$

3. Simplify the following expressions as much as possible. You should show a sequence of equivalent expressions connecting the original expression with your final one, and give a justification for each step (e.g., DeMorgan's law, distributive law, etc.)

   (a) $P \vee (Q \wedge (\neg P \vee R))$

   (b) $(P \vee Q) \wedge \neg(P \vee (Q \wedge \neg P))$

4. Simplify the following expressions as much as possible. You should show a sequence of equivalent expressions connecting the original expression with your final one, and give a justification for each step (e.g., DeMorgan's law, distributive law, etc.)

   (a) $(R \wedge Q) \vee (P \rightarrow (\neg Q \wedge R))$

   (b) $P \leftrightarrow (Q \rightarrow (P \rightarrow Q))$

5. Write useful contrapositives of the following sentences. Express the contrapositives as sentences.

   (a) If the sum of two integers is greater than 100, then at least one of the integers is greater than 25.

   (b) If the morning sky is blue or pink, then the day is warm and pleasant.

   (c) If x and y are integers, and at least one of them is zero, then $xy = 0$.

DeMorgan's laws

$$\neg(P \wedge Q) \text{ is equivalent to } \neg P \vee \neg Q$$

$$\neg(P \vee Q) \text{ is equivalent to } \neg P \wedge \neg Q$$

Commutative Laws

$$P \wedge Q \text{ is equivalent to } Q \wedge P$$

$$P \vee Q \text{ is equivalent to } Q \vee P$$

Associative Laws

$$P \wedge (Q \wedge R) \text{ is equivalent to } (P \wedge Q) \wedge R$$

$$P \vee (Q \vee R) \text{ is equivalent to } (P \vee Q) \vee R$$

Idempotent Laws

$$P \wedge P \text{ is equivalent to } P$$

$$P \vee P \text{ is equivalent to } P$$

Distributive Laws

$$P \wedge (Q \vee R) \text{ is equivalent to } (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \text{ is equivalent to } (P \vee Q) \wedge (P \vee R)$$

Absorption Laws

$$P \vee (P \wedge Q) \text{ is equivalent to } P$$

$$P \wedge (P \vee Q) \text{ is equivalent to } P$$

Double Negation Law

$$\neg\neg P \text{ is equivalent to } P$$

Tautology Laws

$P \wedge$ (a tautology) is equivalent to P

$P \vee$ (a tautology) is a tautology

$\neg$(a tautology) is a contradiction

Contradiction Laws

$P \wedge$ (a contradiction) is a contradiction

$P \vee$ (a contradiction) is equivalent to P

$\neg$(a contradiction) is a tautology

Conditional Laws

$$P \to Q \text{ is equivalent to } \neg P \vee Q$$

$$P \to Q \text{ is equivalent to } \neg(P \wedge \neg Q)$$

Contrapositive Laws

$$P \to Q \text{ is equivalent to } \neg Q \to \neg P$$

Quantifier Negation Laws

$$\neg \exists x P(x) \text{ is equivalent to } \forall x \neg P(x)$$

$$\neg \forall x P(x) \text{ is equivalent to } \exists x \neg P(x)$$

Sets

$$A = B \Leftrightarrow ((x \in A) \Leftrightarrow (x \in B))$$

$$x \in A \cup B \Leftrightarrow ((x \in A) \vee (x \in B))$$

$$x \in A \cap B \Leftrightarrow ((x \in A) \wedge (x \in B))$$

$$x \in A \setminus B \Leftrightarrow (x \in A) \wedge (x \notin B)$$