Math 300 B, C - Spring 2013
Midterm Exam
April 22, 2013

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

1	15	
2	9	
3	10	
4	5	
5	10	
Total	49	

- Complete all five questions.
- You have 50 minutes to complete the exam.

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1. For sets A, B, and C, show the following identities using logic symbols and equivalences.

   (a) $A \setminus (A \cap B) = A \setminus B$

   (b) $(A \setminus B) \setminus C = A \setminus (B \cup C)$

   (c) $(A \setminus B) \cap (B \setminus A) = \varnothing$

2. Write useful contrapositives of each of the following sentences. All variables represent integers.

   (a) If $xy = 3$ and $x < y$, then $x = 1$ and $y = 3$.

   (b) If x is even or y is odd, then $x(y - 1)$ is even.

   (c) If there exists a prime p such that p^2 divides x, then x is not squarefree. (Your contrapositive should incorporate a "for all" statement.)

3. Simplify the following expressions. Justify your results by showing a sequence of equivalent expressions connecting the original expression with your final one.

   (a) $(P \vee Q) \vee \neg(\neg P \vee \neg R)$

   (b) $(P \vee \neg(\neg P \wedge \neg Q)) \wedge \neg((\neg P \wedge R) \vee (R \wedge \neg R))$

4. Write a truth table for the statement $\neg P \wedge (Q \vee P)$

5. Find a formula using only $\neg$ and $\wedge$ that is equivalent to $(P \to Q) \to \neg(Q \vee P)$.

DeMorgan's laws

$\neg(P \wedge Q)$ is equivalent to $\neg P \vee \neg Q$

$\neg(P \vee Q)$ is equivalent to $\neg P \wedge \neg Q$

Commutative Laws

$P \wedge Q$ is equivalent to $Q \wedge P$

$P \vee Q$ is equivalent to $Q \vee P$

Associative Laws

$P \wedge (Q \wedge R)$ is equivalent to $(P \wedge Q) \wedge R$

$P \vee (Q \vee R)$ is equivalent to $(P \vee Q) \vee R$

Idempotent Laws

$P \wedge P$ is equivalent to P

$P \vee P$ is equivalent to P

Distributive Laws

$P \wedge (Q \vee R)$ is equivalent to $(P \wedge Q) \vee (P \wedge R)$

$P \vee (Q \wedge R)$ is equivalent to $(P \vee Q) \wedge (P \vee R)$

Absorption Laws

$P \vee (P \wedge Q)$ is equivalent to P

$P \wedge (P \vee Q)$ is equivalent to P

Double Negation Law

$\neg\neg P$ is equivalent to P

Tautology Laws

$P \wedge$ (a tautology) is equivalent to P

$P \vee$ (a tautology) is a tautology

$\neg$(a tautology) is a contradiction

Contradiction Laws

$P \wedge$ (a contradiction) is a contradiction

$P \vee$ (a contradiction) is equivalent to P

$\neg$(a contradiction) is a tautology

Conditional Laws

$P \to Q$ is equivalent to $\neg P \vee Q$

$P \to Q$ is equivalent to $\neg(P \wedge \neg Q)$

Contrapositive Laws

$P \to Q$ is equivalent to $\neg Q \to \neg P$

Quantifier Negation Laws

$\neg\exists x P(x)$ is equivalent to $\forall x \neg P(x)$

$\neg\forall x P(x)$ is equivalent to $\exists x \neg P(x)$

Sets

$$A = B \Leftrightarrow ((x \in A) \Leftrightarrow (x \in B))$$

$$x \in A \cup B \Leftrightarrow ((x \in A) \vee (x \in B))$$

$$x \in A \cap B \Leftrightarrow ((x \in A) \wedge (x \in B))$$

$$x \in A \setminus B \Leftrightarrow (x \in A) \wedge (x \notin B)$$