Math 300 C - Spring 2015
Midterm Exam Number One
April 22, 2015

Name: ______________________________ Student ID no. : ____________________

Signature: ____

1	10	
2	10	
3	10	
4	5	
5	10	
Total	45	

- Complete all 5 questions.
- You have 50 minutes to complete the exam.

1. Let A, B, and C be sets. Suppose $A \cup C \subseteq B \cup C$. Prove that $A \setminus C \subseteq B$.

2. Let A, B and C be sets. Prove that, if $A \subseteq B \cup C$, then $A = (A \cap B) \cup (A \cap C)$.

3. Let $a, b,$ and c be positive integers. Prove that, if $a|bc$ and $a|(b+c)$, then $a|b^2$.

4. Let $S = \{1, 2, 3\}$. Write out the set $\mathcal{P}(S)$ by listing its elements.

5. Suppose A and B are sets. Suppose $\mathcal{P}(A \setminus B) = \mathcal{P}(A)$. Prove that $A \cap B = \varnothing$.

Axioms of the Integers (AIs)

Suppose a, b, and c are integers.

- **Closure:**

  $a + b$ and ab are integers.

- **Substitution of Equals:**

  If $a = b$, then $a+c = b+c$ and $ac = bc$.

- **Commutativity:**

  $a + b = b + a$ and $ab = ba$.

- **Associativity:**

  $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

- **The Distributive Law:**

  $a(b + c) = ab + ac$

- **Identities:**

  $a + 0 = 0 + a = a$ and $a \cdot 1 = 1 \cdot a = a$

  0 is called the *additive identity*

  1 is called the *multiplicative identity*.

- **Additive Inverses:**

  There exists an integer $-a$ such that $a + (-a) = (-a) + a = 0$.

- **Trichotomy:**

  Exactly one of the following is true: $a < 0$, $-a < 0$, or $a = 0$.

Sets

$A \subseteq B$ iff $x \in A$ implies $x \in B$

$A = B$ iff $A \subseteq B$ and $B \subseteq A$

$x \in A \cup B$ iff $x \in A$ or $x \in B$

$x \in A \cap B$ iff $x \in A$ and $x \in B$

$x \in A \setminus B$ iff $x \in A$ and $x \notin B$

$\mathcal{P}(A)$ is the set of all subsets of a set A

Elementary Properties of the Integers (EPIs)

Suppose a, b, c, and d are integers.

1. $a \cdot 0 = 0$
2. If $a + c = b + c$, then $a = b$.
3. $-a = (-1) \cdot a$
4. $(-a) \cdot b = -(a \cdot b)$
5. $(-a) \cdot (-b) = a \cdot b$
6. If $a \cdot b = 0$, then $a = 0$ or $b = 0$.
7. If $a \leq b$ and $b \leq a$, then $a = b$.
8. If $a < b$ and $b < c$, then $a < c$.
9. If $a < b$, then $a + c < b + c$.
10. If $a < b$ and $0 < c$, then $ac < bc$.
11. If $a < b$ and $c < 0$, then $bc < ac$.
12. If $a < b$ and $c < d$, then $a + c < b + d$.
13. If $0 \leq a < b$ and $0 \leq c < d$, then $ac < bd$.
14. If $a < b$, then $-b < -a$.
15. $0 \leq a^2$
16. If $ab = 1$, then either $a = b = 1$ or $a = b = -1$.

NOTE: Properties 8-14 hold if each $<$ is replaced with $\leq$.

One theorem for reference:

Theorem DAS (Divisors are Smaller): Let a and b be positive integers. Then $a|b$ implies $a \leq b$.