Math 300 A - Winter 2013
Midterm Exam
January 30, 2013
Answers

1. Let A be the statement $(P \vee (Q \wedge P)) \wedge (Q \vee P)$.

   (a) Write out the truth table for A.

P	Q	$Q \wedge P$	$P \vee (Q \wedge P)$	$Q \vee P$	$(P \vee (Q \wedge P)) \wedge (Q \vee P)$
F	F	F	F	F	F
F	T	F	F	T	F
T	F	F	T	T	T
T	T	T	T	T	T

   (b) Write a simpler expression which is equivalent to A.
   We can see from the table that $(P \vee (Q \wedge P)) \wedge (Q \vee P)$ is equivalent to P.

2. (a) Show that, for sets A, B and C,

$$(A \setminus B) \cap C = (A \cap C) \setminus B$$

   using logical symbols and equivalences.

$$\begin{aligned} & x \in (A \setminus B) \cap C \\ \Leftrightarrow\ & x \in A \wedge x \notin B \wedge x \in C \\ \Leftrightarrow\ & (x \in A \wedge x \in C) \wedge x \notin B \\ \Leftrightarrow\ & (x \in A \cap C) \wedge x \notin B \\ \Leftrightarrow\ & x \in (A \cap C) \setminus B. \end{aligned}$$

   Hence, $(A \setminus B) \cap C = (A \cap C) \setminus B$.

   (b) Show that, for sets A, B and C,

$$(B \cup C) \setminus (A \setminus B) = B \cup (C \setminus A)$$

   using logical symbols and equivalences.

$$\begin{aligned} & x \in (B \cup C) \setminus (A \setminus B) \\ \Leftrightarrow\ & (x \in B \vee x \in C) \wedge x \notin (A \setminus B) \\ \Leftrightarrow\ & (x \in B \vee x \in C) \wedge \neg(x \in A \wedge x \notin B) \\ \Leftrightarrow\ & (x \in B \vee x \in C) \wedge (x \notin A \vee x \in B) \\ \Leftrightarrow\ & (x \in B) \vee (x \in C \wedge x \notin A) \\ \Leftrightarrow\ & x \in B \cup (C \setminus A). \end{aligned}$$

   Hence, $(B \cup C) \setminus (A \setminus B) = B \cup (C \setminus A)$.

3. Write a useful negation of each of the following statements using idiomatic mathematical English.

   (a) *For every real number x, there exists a real number y which is greater.*

   There exists a real number x such that, for every real number y, $y \leq x$.

   (b) *Every integer greater than 1 is either prime or composite.*

   There exists an integer greater than 1 which is not prime and not composite.

   (c) *There exists a unique integer n which is divisible by 5 and 7.*

   Either there exists no integer which is divisible by 5 and 7 or there exists more than one integer which is divisible by 5 and 7.

   (d) *Some functions are differentiable.*

   All functions are not differentiable.

4. Find a formula involving only the connectives $\neg$ and $\rightarrow$ that is equivalent to

$$(P \vee Q) \wedge \neg(P \wedge Q).$$

   There are many correct answers. One is

$$\neg((\neg P \rightarrow Q) \rightarrow \neg(P \rightarrow \neg Q))$$

   which can be arrived at like this:

$$\begin{aligned}
&(P \vee Q) \wedge \neg(P \wedge Q) \\
\Leftrightarrow\ &(P \vee Q) \wedge (\neg P \vee \neg Q) \\
\Leftrightarrow\ &\neg(\neg(P \vee Q) \vee \neg(\neg P \vee \neg Q)) \\
\Leftrightarrow\ &\neg((P \vee Q) \rightarrow \neg(\neg P \vee \neg Q)) \\
\Leftrightarrow\ &\neg((\neg P \rightarrow Q) \rightarrow \neg(P \rightarrow \neg Q))
\end{aligned}$$

5. Simplify the expression below as much as you can.

$$\neg(\neg P \wedge Q) \vee (P \wedge \neg R)$$

   This expression can be simplified to $P \vee \neg Q$, or, equivalently, $Q \rightarrow P$.