Math 300 D - Winter 2014
Midterm Exam
January 29, 2014

Name: ______________________________ Student ID no. : ____________________

Signature: ____

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

- Complete all six questions.
- You have 50 minutes to complete the exam.

1. Let A,B,and C be sets. Verify the following identities by showing that that the statement $x \in$ "the left hand set" is equivalent to the statement $x \in$ "the right hand set". Give a justification for each step (e.g., definition of union, DeMorgan's law, distributive law, etc.)

   (a) $A \cap (B \cup (A \cap B)) = A \cap B$

   (b) $(A \cap B) \setminus (B \cap C) = (A \cap B) \setminus C$

2. Let A,B,C and D be sets. Verify the following identity by showing that that the statement $x \in$ "the left hand set" is equivalent to the statement $x \in$ "the right hand set". Give a justification for each step (e.g., definition of union, DeMorgan's law, distributive law, etc.)

$$((A \cup B) \setminus (B \setminus A)) \setminus A = \varnothing$$

3. Simplify the following expressions as much as possible. You should show a sequence of equivalent expressions connecting the original expression with your final one, and give a justification for each step (e.g., DeMorgan's law, distributive law, etc.)

   (a) $P \vee (\neg P \wedge (P \vee Q))$

   (b) $(\neg P \wedge (Q \vee P)) \vee (P \vee \neg Q)$

4. Simplify the following expressions as much as possible. You should show a sequence of equivalent expressions connecting the original expression with your final one, and give a justification for each step (e.g., DeMorgan's law, distributive law, etc.)

   (a) $P \leftrightarrow (Q \rightarrow (P \vee R))$

   (b) $(P \wedge \neg Q) \rightarrow (P \rightarrow Q)$

5. Express the following using only the connectives $\neg$ and $\wedge$:

$$(P \leftrightarrow Q) \vee R$$

6. For a set A, let $\mathcal{P}(A)$ be the power set of A. Write out the set given by the expression

$$\mathcal{P}(\{1, 2, 3\}) \cap \mathcal{P}(\{2, 3, 4\}).$$

DeMorgan's laws

$$\neg(P \wedge Q) \text{ is equivalent to } \neg P \vee \neg Q$$

$$\neg(P \vee Q) \text{ is equivalent to } \neg P \wedge \neg Q$$

Commutative Laws

$$P \wedge Q \text{ is equivalent to } Q \wedge P$$

$$P \vee Q \text{ is equivalent to } Q \vee P$$

Associative Laws

$$P \wedge (Q \wedge R) \text{ is equivalent to } (P \wedge Q) \wedge R$$

$$P \vee (Q \vee R) \text{ is equivalent to } (P \vee Q) \vee R$$

Idempotent Laws

$$P \wedge P \text{ is equivalent to } P$$

$$P \vee P \text{ is equivalent to } P$$

Distributive Laws

$$P \wedge (Q \vee R) \text{ is equivalent to } (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \text{ is equivalent to } (P \vee Q) \wedge (P \vee R)$$

Absorption Laws

$$P \vee (P \wedge Q) \text{ is equivalent to } P$$

$$P \wedge (P \vee Q) \text{ is equivalent to } P$$

Double Negation Law

$$\neg\neg P \text{ is equivalent to } P$$

Tautology Laws

$P \wedge$ (a tautology) is equivalent to P

$P \vee$ (a tautology) is a tautology

$\neg$(a tautology) is a contradiction

Contradiction Laws

$P \wedge$ (a contradiction) is a contradiction

$P \vee$ (a contradiction) is equivalent to P

$\neg$(a contradiction) is a tautology

Conditional Laws

$$P \to Q \text{ is equivalent to } \neg P \vee Q$$

$$P \to Q \text{ is equivalent to } \neg(P \wedge \neg Q)$$

Contrapositive Laws

$$P \to Q \text{ is equivalent to } \neg Q \to \neg P$$

Quantifier Negation Laws

$$\neg\exists x P(x) \text{ is equivalent to } \forall x \neg P(x)$$

$$\neg\forall x P(x) \text{ is equivalent to } \exists x \neg P(x)$$

Sets

$$A = B \Leftrightarrow ((x \in A) \Leftrightarrow (x \in B))$$

$$x \in A \cup B \Leftrightarrow ((x \in A) \vee (x \in B))$$

$$x \in A \cap B \Leftrightarrow ((x \in A) \wedge (x \in B))$$

$$x \in A \setminus B \Leftrightarrow (x \in A) \wedge (x \notin B)$$