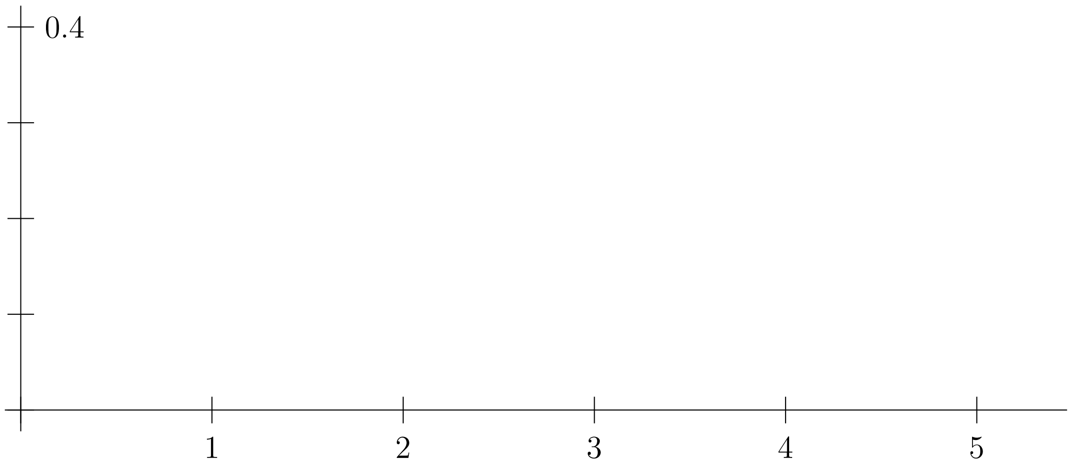
M145 Quiz #01 April 01, 2004

(1a) Let $f(x) = xe^{-x}$. Find the intervals of the positive $x - axis$ where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.

(1b) Find the intervals of the positive $x - axis$ where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.

(1c) What are the maximum and minimum values of $f(x)$ for $0 \leq x \leq 5$?

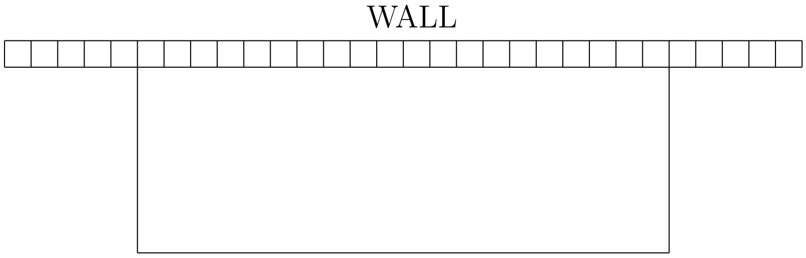
(1d) Sketch the graph of the function $f(x) = xe^{-x}$ for $0 \leq x \leq 5$.



(2a) Solve for t: $e^{-t} - 5e^{-5t} = 0$.

(2b) Solve for x: $3 \ln x + 1 = 0$.

(3) A farmer wants to enclose a rectangular field as shown in the figure. The total area is to ne 1000 m^2. One side is a wall, and needs no fence. The fencing for the sides perpendicular to the wall costs \$ 5 per meter, and the the fencing for the side parallel to the wall costs \$ 4 per meter.



What are the dimensions which minimize the total cost?

M145 Quiz #02 April 08, 2004

(1a) (10pts) Find $f(x)$ if $f'(x) = \dfrac{x^3 + x^2 + x + 1}{\sqrt{x}}$

(1b) (10pts) Find $f(t)$ if $f'(t) = e^{4t} + \cos(5t) + \dfrac{3}{t}$

(2) (30pts) An antibiotic is taken, and t hours after ingestion the serum concentration is

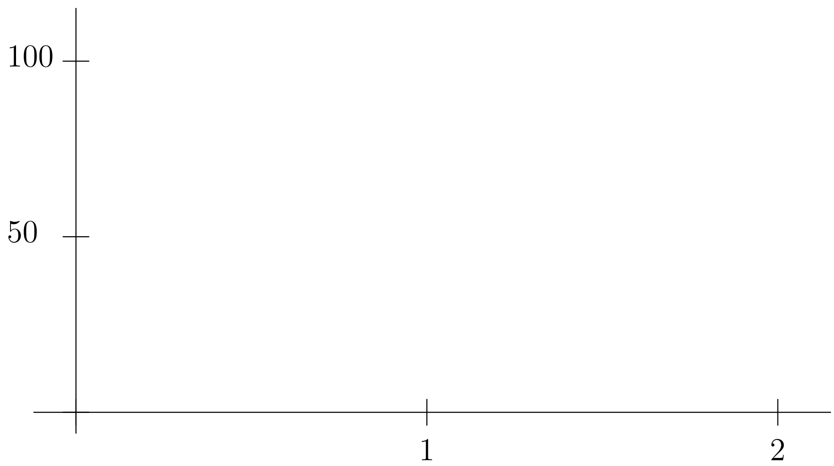
$$C(t) = 100e^{-t} - 100e^{-5t}$$

How long after ingestion will the concentration reach a maximum, and what is the maximum level of concentration achieved?

(3) (20pts) Solve $\dfrac{dy}{dx} = 2x - 1$, given that $y = 4$ when $x = 2$.

(4) (30pts) A box with a square base and no top is to hold 1000 cm^3. The material for the bottom costs \$ 4 per cm^2, and the material for the four sides costs \$ 1 per cm^2. Find the dimensions of the box that minimizes the total cost.

(4b) Sketch the graph of $C(t)$ for $0 \leq t \leq 2$.



M145 Quiz #03 04/15/04

(1) Calculate each of the following antiderivatives.

(1a) $\int (2x+1)^4 dx$

(1b) $\int e^{-\frac{1}{2}x} dx$

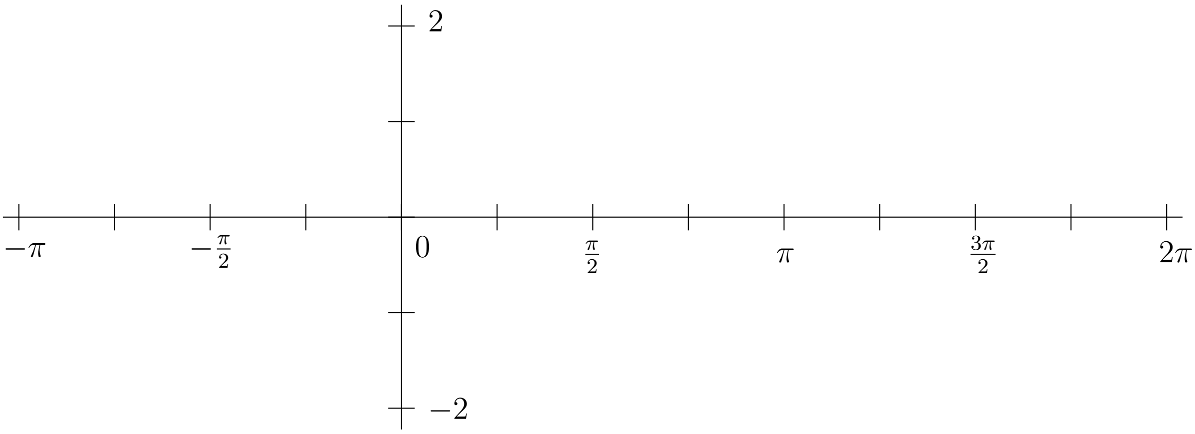
(1c) $\int \left(\frac{x+1+x^{-1}}{x^{\frac{1}{3}}} \right) dx$

(1d) $\int \cos^4(x)\sin(x) dx$

(1e) $\int \left(\frac{x^2}{x+3} \right) dx$ Suggestion: let $u = x+3$

(2a) Evaluate $\int_0^{\pi} (1+\sin(2x))\, dx$

(2b) Sketch the region whose are represents the value of the definite integral of **(2a)**.



(3) An object moves along the x-axis for $t \geq 0$ with acceleration $a(t) = 1+3t$. At time $t = 2$, its velocity is 10, and it is 13 units from the roigin. Where is the object when $t = 4$?

M145 Quiz #03 04/15/04

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(1b) $\int e^{-\frac{1}{2}x} dx$

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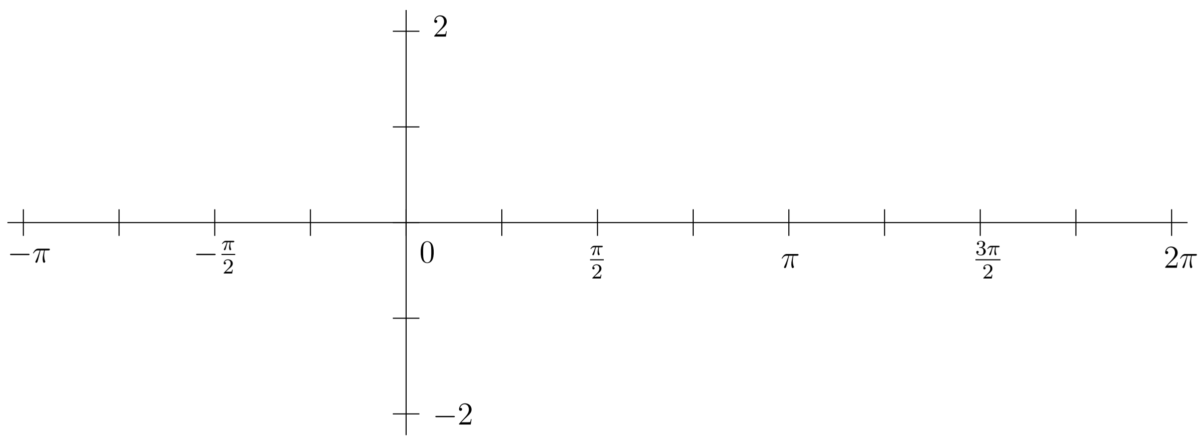

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(2) Evaluate $\int_0^2 \left(\frac{2x}{x^2+3} \right) dx$ Suggestion: let $u = x^2+3$

(3) A population with a limit on its numbers is growing according to the function:

$$N(t) = \frac{100}{1 + 19e^{-0.2\,t}}$$

What is the limit of $N(t)$ as $t \to \infty$?
At what time t does N reach 99% of its limiting value?

(4) An object moves along a line, its velocity given by $v(t)$, where $v(t) = 1 + \sin t$. How far has the object gone when $t = \pi$?

(6) Plant P is growing and for each time t (in days) its height $h(t)$ is measured in cm. P is growing in such a way that its rate of growth (height) is $\left(\frac{1}{4+t} - \frac{1}{10}\right)$ cm per day until it stops growing. Initially (time $t = 0$) its height is 7 cm. What will its height be for each time t? What will its height be when it stops growing?

(2a) Solve $\frac{dy}{dx} = 3x^2 - 2x + 1$, given that $y = 4$ when $x = 2$.

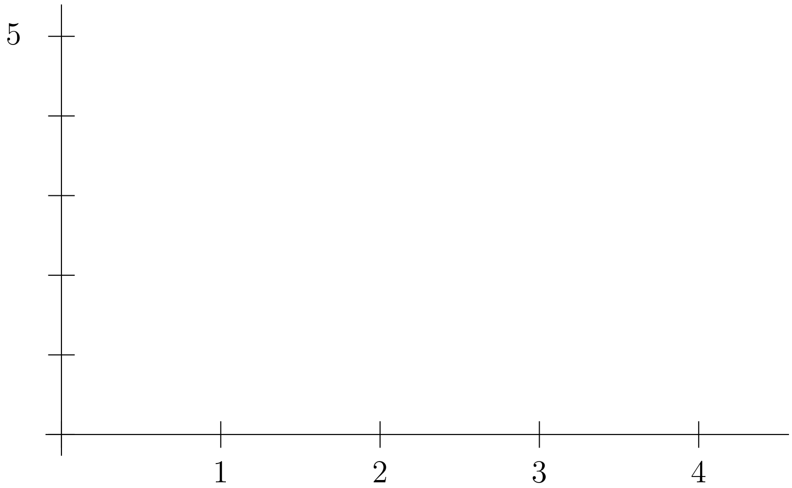
(2a) Solve $y'' = 4$, given that $y(0) = 1$ and $y'(0) = 4$.

M145 Quiz #04 April 22, 2004

(1a) The values of a function at the points $x = 0, 1, 2, 3, 4$ are given in the table:

x	0	1	2	3	4
$f(x)$	1	4	5	4	1

It is known that $f''(x) < 0$ for all x. Sketch a possible function for $0 \leq x \leq 4$ which has these values and satisfies $f''(x) < 0$.



(1b) Subdivide the x-axis for $1 \leq x \leq 4$ by $\Delta x = 1$. Estimate the value of $f(x)$ at the midpoints. Use three rectangles to estimate the definite integral $\int_1^4 f(x)dx$.

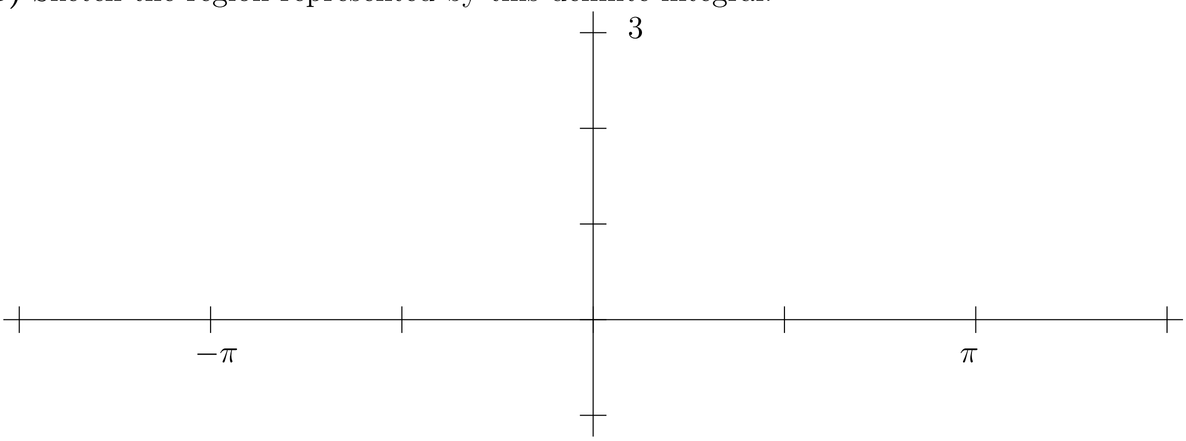
(1c) The function in **(1a)** is $f(x) = -x^2 + 4x + 1$. Use the Fundamental Theorem of Calculus to calculate (exactly) the area under the curve $y = f(x)$ from $x = 1$ to $x = 4$.

(2) $\int (4x + 3)^5 dx =$

(3) Evaluate $\int_0^1 \frac{dt}{e^{-3t} + 1}$ Suggestion: First multiply numerator and denominator by e^{3t}

(4a) Evaluate $\int_{-\frac{\pi}{2}}^{\pi} (2 + \sin(x))\, dx$

(4b) Sketch the region represented by this definite integral.



M145 Quiz #05 April 29, 2004

(2) Evaluate $\displaystyle\int_0^1 \frac{3x^2}{1+x^3}\,dx$

(2) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sin^3\theta\cos\theta d\theta$

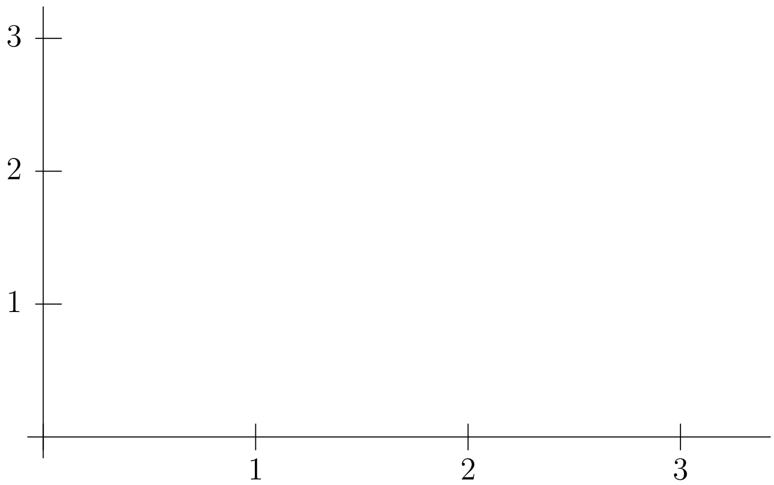
(3) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sec\theta\, d\theta$

(4) Evaluate $\displaystyle\int_1^7 \frac{2e}{1+e^2}\,dx$

(5) Evaluate: $\displaystyle\int_0^1 \frac{dt}{1+2e^{-t}}$.

(6) A plant is growing in such a way that its rate of growth (height) is $h'(t) = \left(\dfrac{1}{2+t} - \dfrac{1}{10}\right)$ cm per day until it stops growing. Initially (time $t=0$) its height is 5 cm. What will its height be when it stops growing?

(7) The curves $y = \dfrac{1}{x}$, and $y = x^2$ and the line $y = 3$ form several regions in the (x, y) plane. Sketch the region that contains the point $(1, 2)$ and calculate its area.



(8) Find the area of the region bounded by the curves $y = x^2$ and $y = (x-2)^2$, and $x = 0$ to $x = 2$.

(10) Evaluate: $\displaystyle\int_0^9 \frac{dt}{1+2e^{-3t}}$. Suggestion: multiply numerator and denominator by e^{3t}.

M145 Quiz #06 May 6, 2004

(1) Evaluate $\int_0^1 \frac{x}{\sqrt{1+x}}\,dx$ Suggestion: Let $u = 1 + x$; then simplify.

(2) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta$ Suggestion: Let $u = 1 + \sin\theta$.

(3) Evalulate $\int_0^2 \frac{dx}{x^2+4}$. Suggestion: Use the substitution $x = 2\tan\theta$.

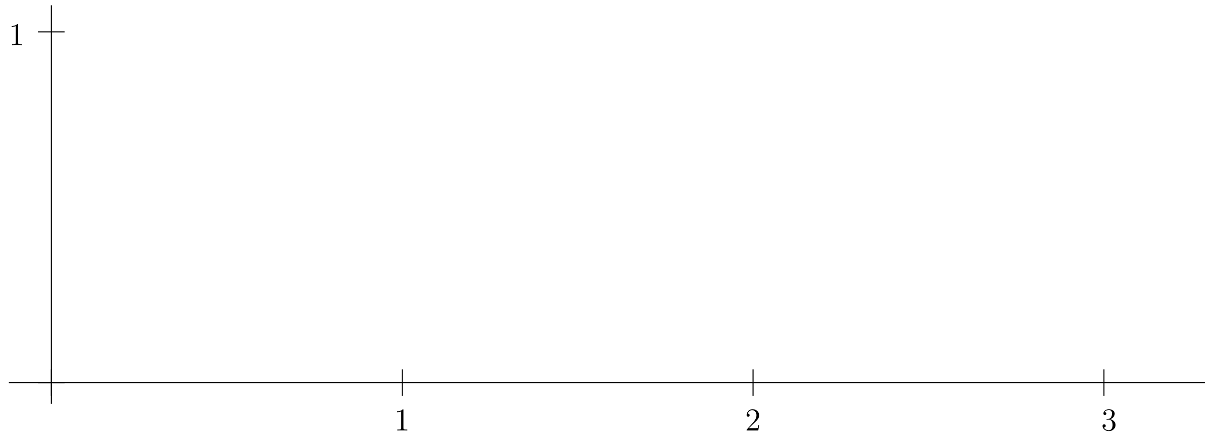
(4) (Measurements in cm.) The line $x = y$ for $0 \le y \le 10$ is rotated about the y-axis to form a cone. There are some particles in the cone: at each height y, the concentration of particles in the cone is $\rho(y) = \frac{1}{1+y^3}$ in billions of particles per cm^3. Calculate the total number of particles in the cone.

(5) For a certain bacteria, the weight in grams at time t in days is

$$y(t) = \frac{20\,dt}{1+3e^{-t}}$$

The bacteria produces toxin at the following rate: Each gram of bacteria makes 0.010 mg of toxin per day. How much toxin is produced in 5 days?

(6) Use the trapezoid rule with two panels to calculate an approximate value for $\int_0^2 \frac{4dx}{x^2+4}$. Sketch the region and the two trapezoids.



M145 Quiz #07 May 13, 2004

(1) (15 pts) Evaluate $\int_0^2 \frac{dx}{\sqrt{4-x^2}}$ Suggestion: Let $x = 2\sin\theta$.

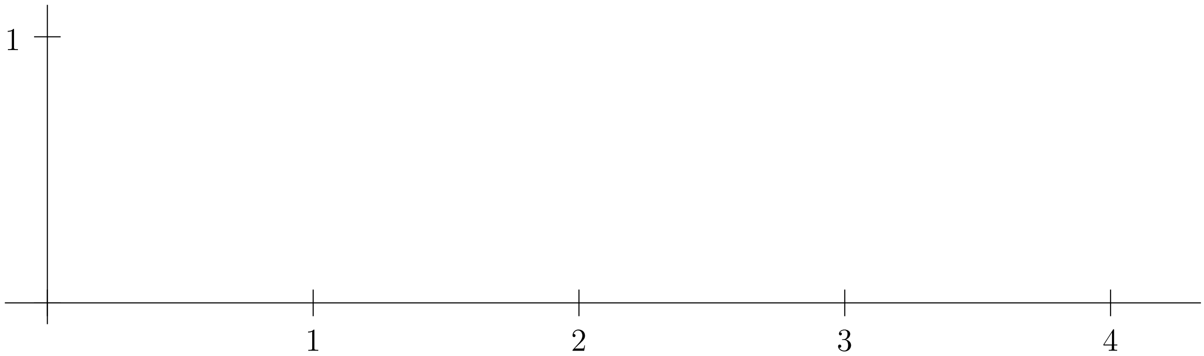
(2) (15 pts) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos\theta\, d\theta}{3-\sin\theta}$ Suggestion: Let $u = 3 - \sin\theta$.

(3) (15 pts) Find the antiderivative: $\int x\cos 5x\, dx$
Suggestion: Int. by Parts: $u = x$; $dv = \cos 5x\, dx$

(4) (15 pts) Find the antiderivative: $\int x^5 \ln x\, dx$
Suggestion: Int. by Parts: $u = \ln x$; $dv = x^5\, dx$

(5) (20 pts) The curve $x = \sqrt{\sin y}$ between $y = 0$ and $y = \frac{\pi}{2}$ is rotated about the y-axis to form a bowl. The bowl is filled with particles. The concentration of particles at height y is $f(y) = \frac{1}{1+\cos y}$ billion particles per unit^3. Calculate the total number of particles in the bowl

(6) (20 pts) Use the trapezoid rule (two trapezoids) to give an approximate value T_2 for $\int_2^4 \ln x\, dx$. Sketch the function $y = \ln x$ for $2 \le x \le 4$ and the trapezoids.



(6b) Is T_2 greater than or less than $\int_2^4 \ln x\, dx$? Explain your answer.

M145 Quiz #08 May 19, 2004

(1) For each of the following improper integrals, determine if it converges or not. If it converges, give the value.

(1a) $\int_0^\infty e^{-3x}\, dx.$

(1b) $\int_1^\infty x^{-\frac{1}{2}}\, dx.$

(1c) $\int_1^\infty x^{-\frac{3}{2}}\, dx.$

(1d) $\int_0^\infty e^{-x} \cos x\, dx$

(2) In the absence of predators, a population of rabbits grows exponentially and would double in 9 months. A pack of foxes eat rabbits at a rate of 7.7 rabbits per month. Initially there are 150 rabbits. Find the number of rabbits for all times t.

(3a) A certain drug is administered intravenously at the rate of 2 mg per hour. The drug is excreted from the body at the (continuous) rate of 20 % per hour. Initially there is no drug in the body. Find the amount of the drug in the body for all times t.

(3b) How long does it take for the amount of the drug in the body to reach 90 % of its limiting value?

M145 Quiz #08a May 24, 2004

(1a) A drug is administered at the rate of 6 mg per hour. The drug is excreted from the body at the (continuous) rate of 30 % per hour (i.e., the intrinsic decay rate is 0.30.) Initially there is no drug in the body. Calculate the amount of the drug in the body for all times t.

(1b) How long does it take for the amount of the drug in the body to reach 95 % of its limiting value?

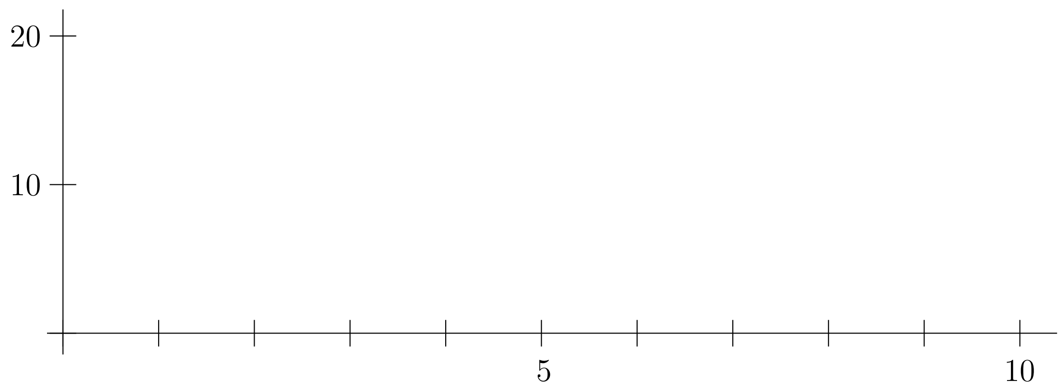
(2a) At midnight 1000 gm of yeast begins to grow exponentially. At 5 AM there are 1649 gm. At noon, an inhibitor is added to the yeast which kills the yeast at the rate of 500 gm per hour. Calculate the amount of yeast at each time t after noon.

(2b) Does all of the yeast eventually die, and if so, at what tome?

M145 Quiz #09 May 27, 2004

(1a) A population of birds grows by a modified logistic growth, called the Allee effect. For this population birds are added at the rate of $y' = 0.1\,y\,(y-1)\,(10-y)$. Find the steady state values and for each steady state value determine whether it is stable or unstable.

(1b) For which (positive) values of the population y will the population increase, and for which (positive) values of the population y will the population decrease? (It might help to graph y' versus y.)



(2a) Solve the differential equation: $\dfrac{dy}{dt} = y(1 - \dfrac{y}{10})$ with $y(0) = 2$. Express your answer in the form $y = \dfrac{K}{1 + ae^{-rt}}$ for suitable constants K, a and r.

(2b) For which value of y is $\dfrac{dy}{dt}$ a maximum, and what is this maximum value?

M145 Quiz #10 June 5, 2004

(1) $y = f(t)$ is a function of t which satisfies $y' = 0.1y(y-5)(40-y)$. Without finding the explicit solution to this differential equation, answer the following.

(1a) What are the critical values. For each critical value, state whether or not it is stable.

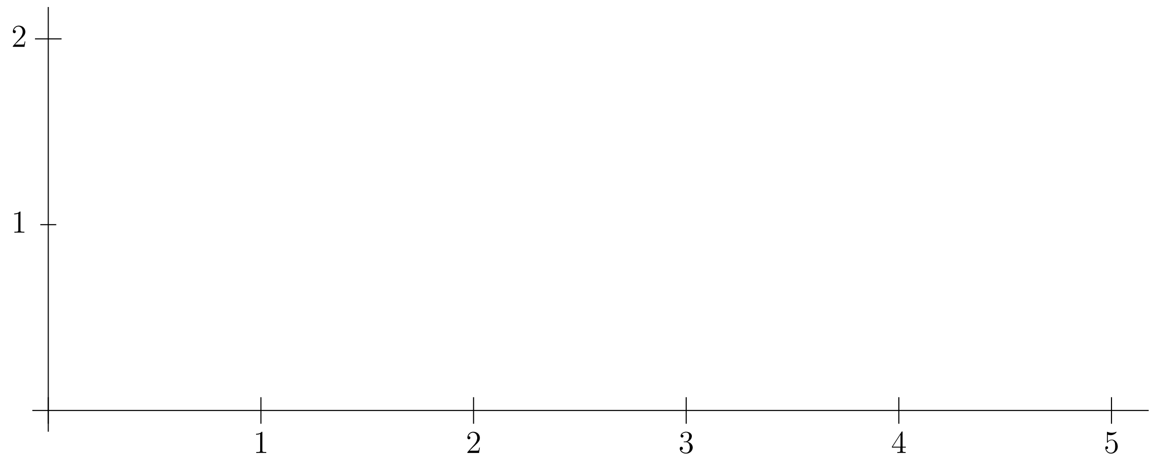
(1b) If $f(0) = 4$, what is $\lim_{t\to\infty} f(t)$?

(1c) If $f(0) = 10$, what is $\lim_{t\to\infty} f(t)$?

(1d) If $f(0) = 100$, what is $\lim_{t\to\infty} f(t)$?

(2) Evaluate $\int_0^\infty xe^{-2x}\,dx$. Suggestion: Int. by Parts: Let $u = x$, $dv = e^{-2x}\,dx$, etc

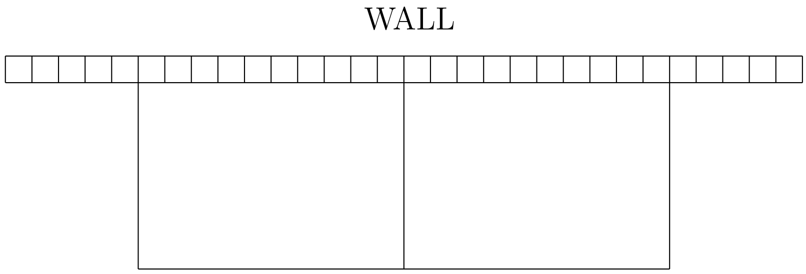
(3a) Let R be the region in the first quadrant bounded by the lines $y = 0$, $x = 5$, and the curve $y = \sqrt{x-1}$. Sketch the region R.



(3b) Express the area of R as a definite integral in **TWO** ways: (1) with x as the variable of integration; and (2) with y as the variable of integration.

(3c) Evaluate **BOTH** definite integrals of **(3b)**.

(4) A farmer wants to enclose two fields with total area 1200 square meters as follows. One side of the field is a wall and needs no fence, one side is parallel to the wall, and there are three sides perpendicular to the wall, making two equal fields as shown in the figure. The side parallel to the wall costs \$1 per meter, and the sides perpendicular to the wall cost \$4 per meter. What dimensions minimize the total cost?



(5) A cell has a quantity of protein molecules which are continuously lost and replenished. New protein is made at the rate of 10 molecules per day and the loss rate is 20 % per day (that is the continuous loss rate is 0.2). After an infection the number of protein molecules is 30. How long will it take (after the infection ends) for the number of protein molecules to return to 90 % of its steady state value?