M145 Quiz #01a Jan 7, 2004

Calculate each of the following antiderivatives.

(1) $\int (x^3 - x^2 + x - 1)\, dx$

(2) $\int \left(e^{3x} + e^{-\frac{1}{2}x}\right)\, dx$

(3) $\int \left(\frac{x + 1 + x^{-1}}{\sqrt{x}}\right)\, dx$

(4) $\int (\sin(3x) + \cos(4x))\, dx$

(5) $\int \left(\frac{x^3 + x^2 + x + 1}{x^2}\right)\, dx$

M145 Test #01b Jan 11, 2005

(1) Find the antiderivative: $\int \cos^5 x \cdot \sin x dx$

(2) Find the antiderivative: $\int_1^2 x\sqrt{3 + x^2}\, dx$

(3) Evaluate $\int_0^1 \frac{x^2}{x^3 + 1}\, dx$

(4) Evaluate $\int_0^1 \frac{1}{1 + e^{-t}} dt$

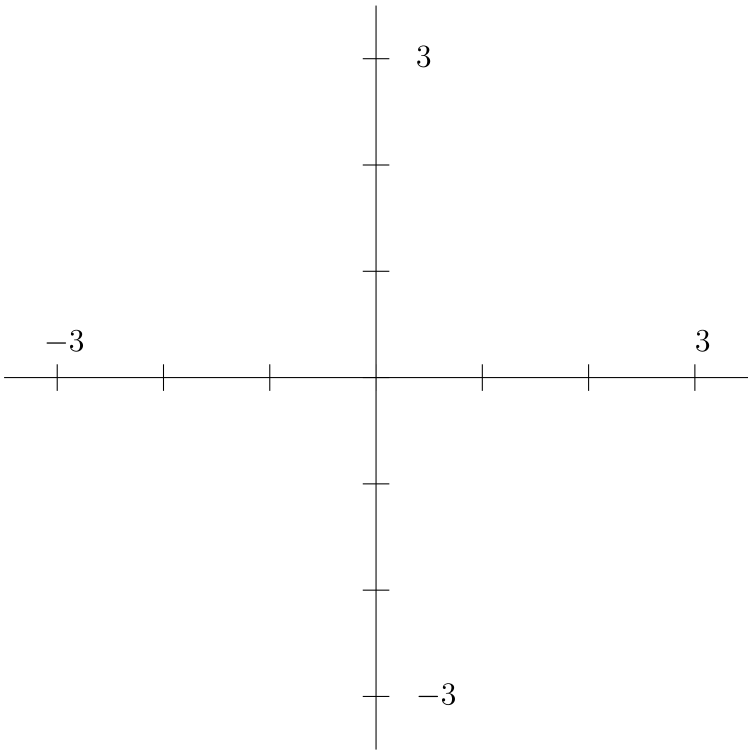
M145 Test #03 Jan 18, 2005 **SHOW WORK**

(1) Evaluate $\displaystyle\int_0^{\pi} \frac{\sin x}{\sqrt{2+\cos x}}dx$

(2) Solve $f''(x) = 6x + 2$, given that $f(1) = 9$ and $f'(1) = 10$.

(3) Evaluate $\displaystyle\int_0^{3} \frac{x}{3+x^2}dx$

(4) Calculate the area between the curves $y = 3 - x^2$ and the line $x + y = 1$. You might want to sketch this region.



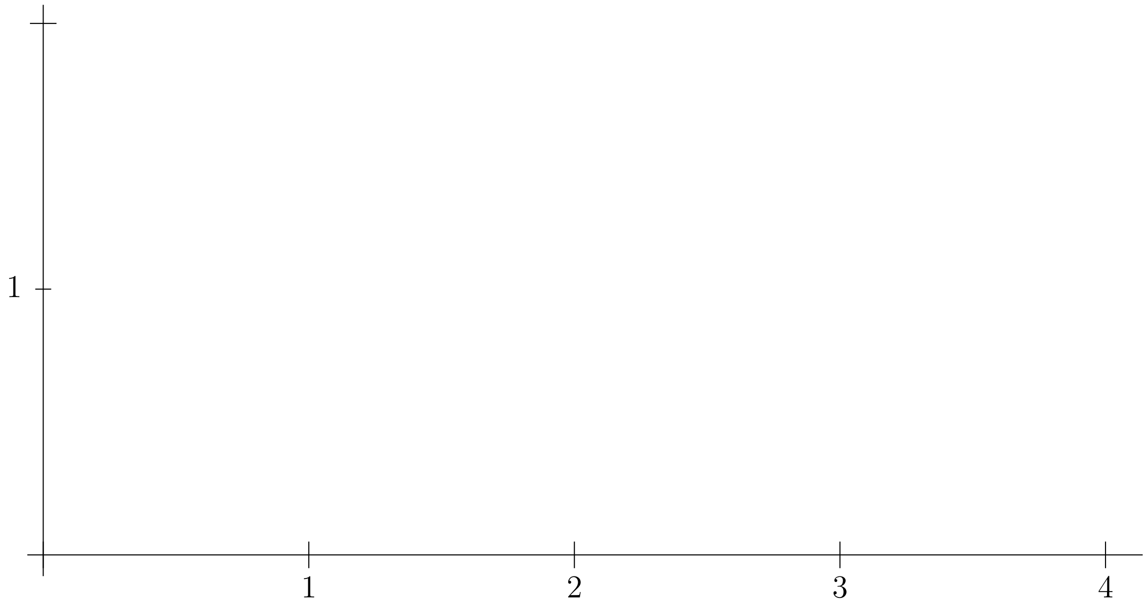
M145 Test 04 Jan 25, 2005 **SHOW WORK**

(1) (20 pts) Evaluate $\int_0^3 \left(\frac{2x}{x^2+1}\right) dx$ Suggestion: Substitution $u = x^2 + 1$

(2) (20 pts) Evaluate $\int_0^1 \frac{dt}{e^{-3t}+1}$ Suggestion: Multiply numerator and denominator by e^{3t}

(3) (20 pts) Evaluate $\int_2^5 \frac{x dx}{\sqrt{x-1}}$ Suggestion: Substitution $u = x - 1$.

(4) (40 pts) The curves $y = \frac{1}{x}$, $x = y^2$ and the straight line $y = \frac{x}{3}$ separate the first quadrant into several regions. Calculate the area of the region which contains the point $(1, \frac{1}{2})$. You might want to sketch the region below:



M145 Test #05 Feb 1, 2005

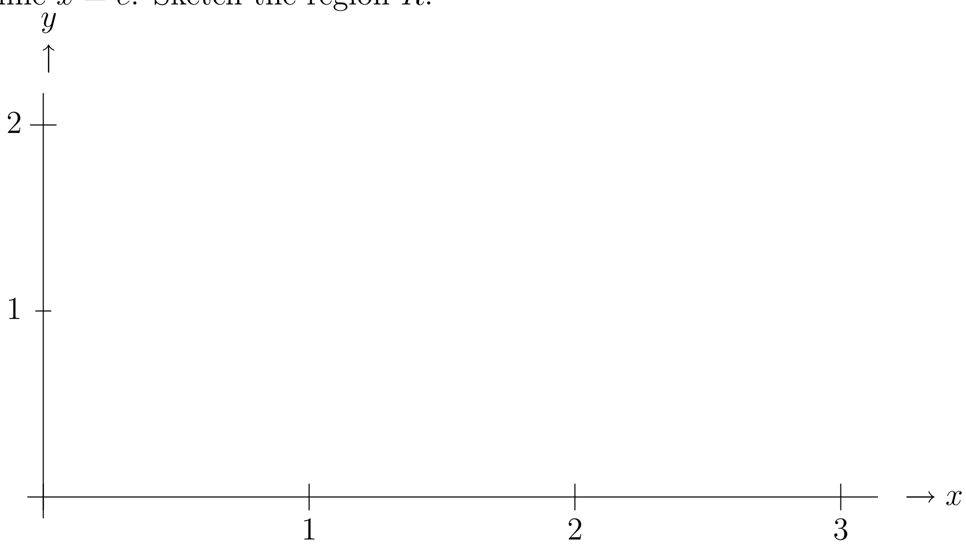
(1) Use the trapezoid rule with $\Delta x = 1$ to give an approximate value for the area under the curve $f(x) = \dfrac{x}{1+x}$. from $x = 1$ to $x = 5$.

(1b) Is the approximate value obtained by the trapezoid rule for **(1a)** too high or too low.Explain.

(2) Use the Fundamental Theorem of Calculus to calculate the area under the curve $f(x) = \dfrac{x}{1+x}$. from $x = 1$ to $x = 5$. Suggestion: substitution $1 + x = u$.

(3) Use Simpson's Rule with $\Delta x = 1$, to to give an approximate value for the area under the curve $f(x) = \dfrac{1}{1+\sqrt{x}}$. from $x = 0$ to $x = 4$.

(4) Let R be the region in the first quadrant bounded by the curve $y = \ln x$, the x-axis and the line $x = e$. Sketch the region R.



Express the area of R as a definite integral in **TWO** ways:
(1) with x as the variable of integration; and
(2) with y as the variable of integration. Evaluate either integral.

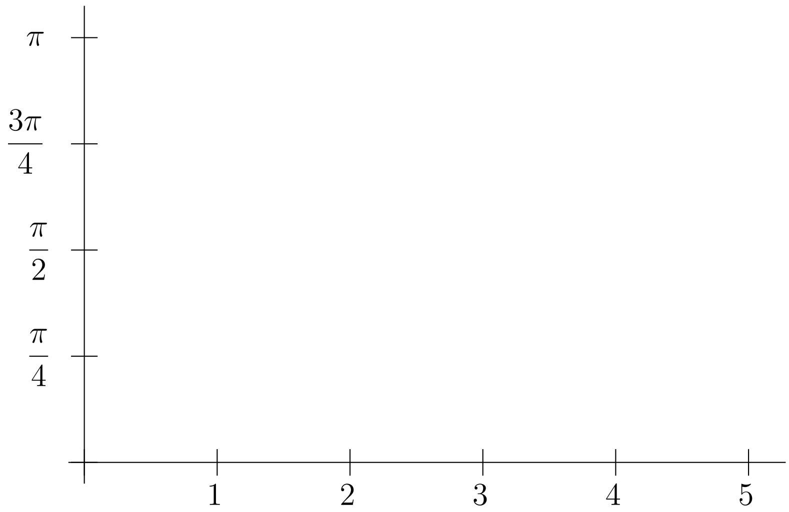
M145 Test #06 Feb 8, 2005 **SHOW WORK**

(1) Evaluate $\int_0^{\frac{\pi}{2}} x \cos x \, dx$

(2) Evaluate $\int_0^1 \frac{x \, dx}{x^4 + 1}$ Suggestion: start by letting $x^2 = u$

(3) Evaluate $\int_6^8 \sqrt{100 - x^2} \, dx$

(4) Let R be the region in the first quadrant bounded by the curve $y = \arcsin \frac{x}{5}$, the x-axis and the line $x = 5$. You might want to sketch the region R.



Express the area of R as a definite integral in **TWO** ways:

(a) with x as the variable of integration; and

(b) with y as the variable of integration.

(c) Evaluate both integrals

M145 Test #07 Feb 15, 2005 **SHOW WORK**

(1) Evaluate: $\int_1^4 \frac{dy}{y(5-y)}$

(2) A drug is administered at the rate of 6 mg per hour. The drug is excreted from the body at the (continuous) rate of 30 % per hour (i.e., the intrinsic decay rate is 0.30). Initially there is no drug in the body. Calculate the amount of the drug in the body for all times t.

(2b) At what time does the amount of the drug in the body to reach 95 % of its limiting value?

(3) A body is infected by bacteria. At 12 noon, there are 10 billion bacteria. At 4 PM there are 40 billion bacteria, at which time a drug is administered which kills the bacteria at the rate of 25 Billion per hour. Find the number of bacteria present at each time t.

(3b) Does the drug eventually kill all of the bacteria, and if so, at what time are all the bacteria dead?

(4a) Find the antiderivative: $\int \frac{10}{y(10-y)} dy$

(4b) Solve the equation $\frac{dy}{dt} = y(1 - \frac{y}{10})$, with $y(0) = 1$.

(5a) Find the antiderivative: $\int \frac{100}{y(100-y)} dy$

(5b) Solve the equation $\frac{dy}{dt} = 0.5y(100 - y)$, with $y(0) = 1$.

M145 Test #08 Feb 22, 2005

(1a) (1) Cobalt-60 is a radioactive element with a half-life of 5.3 years. Let $f(t)$ be the probability density function for the decay time of an atom of Cobalt-60. Calculate $f(t)$.

(1b) Calculate the mean μ for $f(t)$.

(1c) If you have an atom of Co-60 today, what is the probability that it will decay some time in the next three years?

(2a) A bacteria Y has a growth rate given by the equation: $\dfrac{dy}{dt} = 0.01y(20 - y)$
Initially there are 2 grams of Y. Find the amount $y(t)$ for all times t.
Express your answer in the form $y = \dfrac{K}{1 + ae^{-rt}}$ for suitable constants K, a and r.

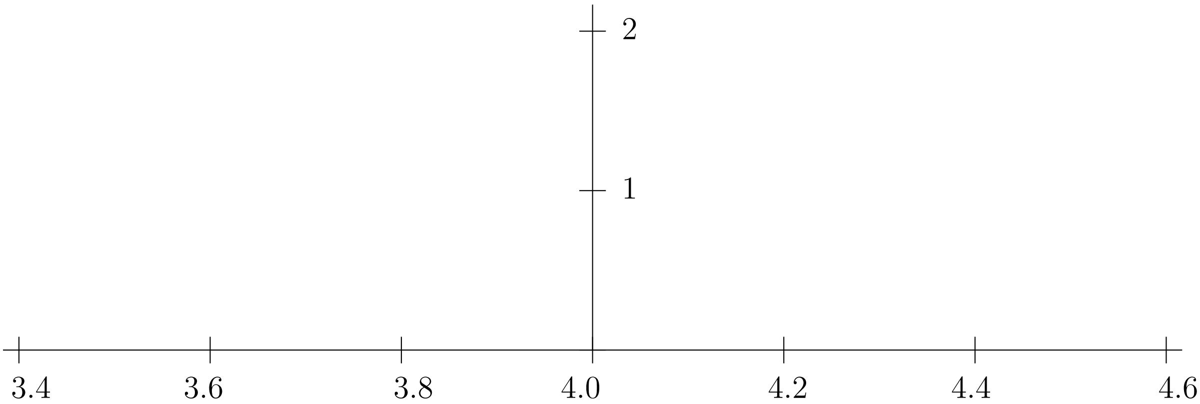
(2b) For which value of y is $\dfrac{dy}{dt}$ a maximum, and at what time does it occur?

(3) Suppose y is the function oftime t given by $y = f(t) = \dfrac{40}{1 + 3e^{-0.2t}}$. At what time does $y(t)$ reach 95 % of its limiting value?

M145 Test #09 March 1, 2005

(1a) The lengths of fish in a certain population is normally distributed, with a mean of 4.0 cm and a standard deviation of 0.2 cm. What fraction of the fish have a length that lies between 3.8 cm and 4.3 cm?

(1b) Let $F(x)$ be the probability density function for **(1a)**. Sketch $F(x)$ and sketch the region which represents the fraction of the fish of **(1)** that have a length between 3.8 cm and 4.3 cm.



Place the points corresponding to $F(3.4)$, $F(3.6)$, $F(3.8)$, $F(4.0)$, $F(4.2)$, $F(4.4)$, $F(4.6)$ as accurately as you can. Be careful with the vertical scale.

Note: For the probability density function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, some values are

$f(\pm 3) \approx 0.004$ $\quad f(\pm 2) \approx 0.054$ $\quad f(\pm 1) \approx 0.24$ $\quad f(0) \approx .4$

(2) In a population, there are two blood tupes A and B. The probability of A is .64; the probability of B is .36 If you select 25 memmbers at random, what is the probability that the number of type A is 15, 16, 17 or 18?? (Use the histogram correction)

(3) The heights of plants are normally distributed. Half of the plants are shorter than 50 cm, and 85% of them are shorter than 55.7 cm. What fraction are shorter than 52 cm?

M145 Test #10 March 8, 2005

(1a) Batteries have an average life of 5 years. What fraction of the batteries will last 8 years?

(1b) If you buy a battery today, what is the probability that it will die in the first year?

(2) The weights of girls at age 11 are normally distributed. Half of them weigh less than 80 lbs, and 90 % weigh less than 100 lbs. How many weigh less than 70 lbs?

(3) A certain gene has 10^6 sites where a mutation might occur. For each site, the probability of a mutation is $1.5 \cdot 10^{-6}$. What is the probability that there are eactly two mutations in the gene?

(4) In the US population, a certain disease occurs in 3% of the population. In a sample of 100 people, what is the probability that three or more have the disease?

M145 Test #11 March 12, 2005

(1a) Evaluate $\int_2^3 \left(\frac{x^2+x}{x-1}\right) dx$ Suggestion: let $u = x - 1$

(1b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos t dt}{1+\sin t}$ Suggestion: let $u = 1 + \sin t$

(1c) Evaluate $\int_2^4 \frac{5dy}{y(5-y)}$

(2) Atoms of Barium 140 have a half-life 13 days. If you have an atom of Ba-140 today, what is the probability that it will decay in the next 5 days?

(3) You have a biased coin that comes up heads 60 % of the time. The coin is tossed 24 times. What is the probability that the number of heads is 13, 14, 15, 16 or 17. (Use the normal approximation to the binomial distribution, and the histogram correction.)

(4) In the absence of predators, a population of rabbits grows exponentially and would double in 3 months. A pack of foxes eat rabbits at a rate of 46.2 rabbits per month. Initially there are 300 rabbits. Find the number of rabbits for all times t.