MATH 145 **Winter 2006**

The Exponential Distribution

The Exponential Density Function with parameter λ is $f(t) = \lambda e^{-\lambda t}$. For the function $e^{-\lambda t}$,

$$\int_0^\infty e^{-\lambda t} dt = -\left[\frac{1}{\lambda} e^{-\lambda t}\right]_0^\infty = \frac{1}{\lambda}$$

Thus, $e^{-\lambda t}$ is not a probability density function (unless $\lambda = 1$), so we multiply by λ and

$$\int_0^\infty \lambda e^{-\lambda t} dt = 1$$

Suppose that an event can occur at any time T between $t = 0$ and $t = \infty$. We say that the times (of the event) are exponentially distributed with parameter λ if the probability that T is between a and b is

$$P(a \leq T \leq b) = \int_a^b \lambda e^{-\lambda t} dt = \Big[-e^{-\lambda t}\Big]_a^b = -e^{-\lambda b} + e^{-\lambda a}$$

The Mean is $\mu = \dfrac{1}{\lambda}$.

The Half-life is $H = \dfrac{\ln 2}{\lambda}$

Example The arrival tines of a bus are exponential distributed with parameter $\lambda = 0.2$. The probability that a bus will arrive in the next 10 minutes is

$$P(0 < T < 10) = \int_0^{10} 0.2e^{-0.2t} dt = -e^{-(0.2)(10)} + e^{-0} = -e^{-2} + 1 = -.135 + 1 \approx .865$$

The average length of time to wait for a bus is $\mu = \dfrac{1}{.2} = 5$ minutes.

(2) The average age of lightbulbs is 2000 hours. Assume that the lifetimes that are exponentially distributed.

(a) What is the half life of a bulb?

(b) What is the probabability density function?

(c) What is the probability that a bulb is still working after 2000 hours?

Solution (1) (With one thousand hours as the unit of time.)

(a) We are given that $\mu = 2$, so $\lambda = \dfrac{1}{2} = .5$ $H = \dfrac{.693}{.5} \approx 1.386$ thousand hours.

(b) $f(t) = (0.5)e^{-0.5t}$.

(c) $P(2 \leq T < \infty) = \displaystyle\int_2^\infty \lambda e^{-0.5t} dt = \Big[-e^{-0.5t}\Big]_2^\infty = -0 + e^{-1} \approx .368$