

**M145 Quiz #01** Jan 10, 2006 **SHOW ALL WORK**

(1)  $\int (x + 1)^4 dx =$

(2)  $\int (x^2 + 1)^5 \cdot 2x dx =$

(3)  $\int e^{-\frac{x}{2}} dx =$

(4) Evaluate  $\int_0^\pi \cos\left(\frac{x}{2}\right) dx$

(5) Evaluate  $\int_1^4 \frac{1}{x+2} dx$

(6)  $\int \frac{x^2 + 1 + x^{-1}}{\sqrt{x}} dx =$

(7) Solve  $\frac{dy}{dx} = 3x^2 - 2x + 1$ , given that  $y = 4$  when  $x = 1$ .

(8) Calculate the area under the curve  $y = \frac{1}{x^3}$  from  $x = 1$  to  $x = 4$ .

**M145 Test #02** Jan 17, 2006 **SHOW ALL WORK**

(1)  $\int \frac{x^2 + x + 1}{x + 1} dx =$

(2) Evaluate  $\int_1^3 \frac{x dx}{\sqrt{3 + x^2}} =$

(3) Evaluate  $\int_0^1 \frac{e^t dt}{3e^t + 1} =$

(4) Evaluate  $\int_0^{\frac{\pi}{2}} \sin^2 \theta \cos \theta d\theta$

(5) Find the antiderivative :  $\int (\sin(3\theta) + \cos(5\theta)) d\theta$

(6) Find the antiderivative:  $\int e^{\sin x} \cos x dx$

(7) Solve  $y'' = 4$ , given that  $y = 1$  and  $y' = 6$  when  $x = 1$ .

(8) Calculate the area bounded by the lines  $y = 0$ ,  $x = 1$ ,  $x = 3$ , and the graph of  $y = \frac{1}{x}$

**M145 Quiz #03** Jan 24, 2006 **SHOW ALL WORK**

(1) Evaluate  $\int_1^2 \frac{dt}{t\sqrt{1+\ln t}}$ . Suggestion: let  $u = 1 + \ln t$ .

(2a) Use the trapezoid rule with  $\Delta x = 1$  (three panels) to calculate an approximate value for  $\int_1^4 \frac{dx}{1+x}$ . Sketch the region and the trapezoids.

(2b) Is the estimate of (2a) for the value of  $\int_1^4 \frac{dx}{1+x}$  too high or too low? Give reasoning.

(3) Use the Fundamental Theorem of Calculus to calculate  $\int_1^4 \frac{dx}{1+x}$ .

(4) The curve  $y = \frac{1}{x}$ , the line  $y = x$ , and the line  $y = 2$  form several regions in the  $(x, y)$  plane.  $R$  is the region that contains the point  $(1, 1.5)$ .

Express the area of  $R$  as a definite integral in TWO ways and then evaluate BOTH integrals.

(a) With  $x$  as the variable of integration; and

(b) With  $y$  as the variable of integration.

**M145 Test #04** Jan 31, 2006 **SHOW ALL WORK**

(1) Evaluate  $\int_{x=2}^{x=7} \frac{x \, dx}{\sqrt{x+2}}$

(2a) Evaluate  $\int_1^e \frac{dt}{t(1+\ln t)}$ .

(2b) Use the trapezoid rule with one panel to calculate an approximate value for the integral of (1a).

(3) A function  $y = f(x)$  for  $0 \leq x \leq 4$  is rotated about the  $x$ -axis to form a solid object. Measurements of the radii of cross-sections of the object are made and recorded in the table:

$x$	0	1	2	3	4
radius of cross-section at $x$	2	3	4	5	4

Use Simpson's rule with  $\Delta x = 1$  to estimate the volume of the object.

(4) An object moves along the  $x$ -axis for  $t \geq 0$ . The initial position is  $x(0) = 0$ , the initial velocity is 0, the acceleration is  $a(t) = 1 - (1+t)^{-2}$ . How far has the object gone when  $t = 2$ ?

(5) The curve  $\sin y = x^2$  for  $x = 0$  to  $x = 1$  is rotated about the  $y$ -axis to form a bowl. Calculate the volume of the bowl.

(6) The curve  $y = x^4$  for  $x = 0$  to  $x = 2$  is rotated about the  $y$ -axis to form a bowl.

(6a) Express the volume inside bowl with  $y$  as the variable of integration.

(6b) Calculate the volume inside the bowl

**M145 Quiz #05** Feb 7, 2006 **SHOW ALL WORK**

(1) Evaluate  $\int_{x=1}^{x=3} x^2 \ln x \, dx$

(2) Evaluate  $\int_{t=0}^{t=1} \frac{dt}{1 + 2e^{-t}}$  Suggestion: multiply numerator and denominator by  $e^t$

(3) Evaluate  $\int_{x=0}^{x=1} e^{\sqrt{x}} \, dx$

(4) Evaluate:  $\int_{x=3}^{x=4} \sqrt{25 - x^2} \, dx$

(5) Let  $R$  be the region in the first quadrant bounded by the curve  $x = y^2 + 1$ , the  $x$ -axis and the line  $x = 5$ .

Express the area of  $R$  as a definite integral in **TWO** ways; Evaluate both integrals.

(5b) With  $x$  as the variable of integration.

(5c) With  $y$  as the variable of integration.

**M145 Quiz #06** Feb 14, 2006 **SHOW ALL WORK**

(1) The curves  $y = \frac{1}{x}$ ,  $x = y^3$  and the straight line  $y = \frac{x}{4}$  separate the first quadrant into several regions. (how many are there?) Calculate the area of the region which contains the point  $(1, 0.5)$ .

(2) Evaluate  $\int_{10}^{11} \frac{dt}{t(t-5)}$

(3) In the absence of predators, a population of rabbits grows exponentially and would double in 9 months. A pack of foxes eat rabbits at a rate of 7.7 rabbits per month. Initially there are 120 rabbits. Write a rate equation for the number  $y(t)$  of rabbits at each time  $t$ .

(3b) Solve this differential equation to find the number of rabbits for all times  $t$ .

(3c) Which happens? (A) All rabbits get eaten.

OR (B) The population of rabbits increases without limit.

If answer is (A), at what time does it occur?

If answer is (B), at what time does the population reach twice its original size?

**M145 Quiz #07** Feb 21, 2006 **SHOW ALL WORK**

(1a) At midnight 1000 gm of yeast begins to grow exponentially. At 4 AM there are 1492 gm. How much yeast is present at 12 noon?

(1b) At 12 noon, an inhibitor is added to the yeast which kills the yeast at the rate of 500 gm per hour. Calculate the amount of yeast at each time  $t$  after noon.

(1c) Does all of the yeast eventually die, and if so, at what time?

(2a) A brand of transistor has an average life of 20 hours. What is the half-life of these transistors?

(2b) What is the probability that a transistor will last 40 hours?

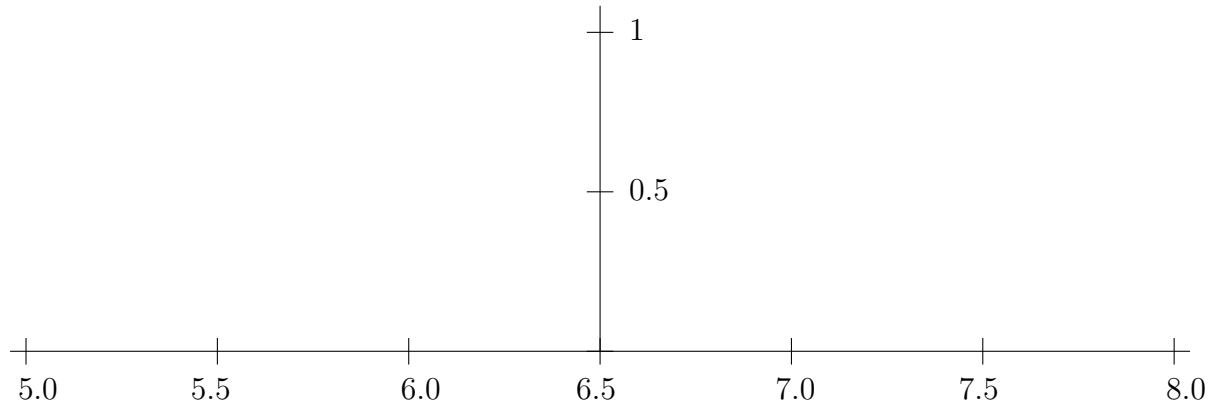
(3) Barium-140 has a half life of 13 days. If you have an atom of Ba-140 at day 0, what is the probability that it will decay some time between day 4 and day 17?

(3b) What is the average lifetime of Ba-140 atoms?

M145 Test #08 Feb 28, 2006 SHOW ALL WORK

(1a) The heights of a population of plants are normally distributed, with a mean of 6.5 cm and a standard deviation of 0.5 cm. What fraction of heights are between 5.5 and 7.0?

(1b) Let  $F(x)$  be the probability density function for these heights. Sketch  $F(x)$  and sketch the region which represents the fraction of individuals that have a height between 5.5 cm and 7.0 cm.



Place the points corresponding to  $F(5.0)$ ,  $F(5.5)$ ,  $F(6.0)$ ,  $F(6.5)$ ,  $F(7.0)$ ,  $F(7.5)$ ,  $F(8.0)$  as accurately as you can. For the function  $f(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ , some values are

$$f(\pm 3) \approx 0.004 \quad f(\pm 2) \approx 0.054 \quad f(\pm 1) \approx 0.24 \quad f(0) \approx .4$$

(2) The scores on an exam are normally distributed. What is the probability that a random score  $S$  is within 1 standard deviation of the mean?

(3) In a population, there are two blood types A and B; 64% of the individuals have type A and 36% have type B. If 25 samples are selected at random, what is the probability that the number with type A is 15, 16, 17 or 18?? (Use the normal approximation to the binomial distribution, and use histogram correction)

(4) The heights of plants are normally distributed; 50 % are shorter than 25 cm and 75% are shorter than 27.7 cm. What is the probability that the height of a plant is between 21 and 27 cm?

**M145 Quiz #09** Feb March 7, 2006 **SHOW ALL WORK**

(1a) Evaluate  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{4n}$

(1b) Evaluate  $\lim_{n \rightarrow \infty} \left(1 - \frac{1.5}{n}\right)^{2n}$

(1c) Evaluate  $\lim_{t \rightarrow \infty} \left(\frac{100e^t}{2e^t + 5e^{-t}}\right)$

(2a) A certain nucleus contains  $10^6$  genes. Each gene has probability  $1.2 \times 10^{-6}$  of being mutated. What is the probability that none of these genes are mutated?

(2b) What is the probability that two or fewer of these genes are mutated?

(3a) A switchboard receives an average of 90 calls per hour. What is the probability that it receives exactly 2 in the next two minutes?

(3b) What is the probability that the switchboard of (3a) receives 2 or more calls in the next two minutes?

(4) 16 % of a population have blood type X. In a random sample of 100 individuals, what is the probability that the number with type X is 16, 17, 18, or 19? (Use the histogram correction.)

**M145 Final Exam** March 11, 2006 **SHOW ALL WORK**

(1a) Evaluate  $\int_0^2 x^2 \sqrt{1+x^3} dx$

(1b) Evaluate  $\int_0^1 x e^{-x} dx$

(1c) Evaluate  $\int_0^\pi \cos \theta \sqrt{1+\sin \theta} d\theta$

(2a) Evaluate  $\lim_{t \rightarrow \infty} \left( \frac{100}{2+3e^{-5t}} \right)$

(2b) Evaluate  $\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{2n} \right)^{3n}$  ?

(3) Let  $R$  be the region above the  $x$ -axis and below the curve  $y = \ln x$  for  $x = 1$  to  $x = 6$ . Express the area of  $R$  as a definite integral in TWO ways and then evaluate BOTH integrals.

(3a) With  $x$  as the variable of integration; and

(3b) With  $y$  as the variable of integration.

(4) A function  $y = f(x)$  for  $0 \leq x \leq 4$  is rotated about the  $x$ -axis to form a solid object. Measurements of the circumference of cross-sections of the object are made and recorded in the table:

$x$	0	1	2	3	4
circumference at $x$	12	20	30	36	32

Use the trapezoid rule with four panels ( $\Delta x = 1$ ) to estimate the volume of the object.

(5) A pea has a .64 probability of being smooth, and a .36 probability of being wrinkled. If you select 100 peas at random, what is the probability that at least 60 and at most 70 are smooth? (Use the normal approximation to the binomial distribution, with the histogram correction)

(6a) At 9 AM, 100 gm of yeast begins to grow exponentially. At 11 AM there are 150 gm. Calculate the amount of yeast at noon.

(6b) At noon, an inhibitor is added which kills the yeast at the rate of 40 gm per hour. Calculate the amount of yeast at each time  $t$  after noon.

(6c) Does all of the yeast eventually die, and if so, at what time?

(7a) Cobalt<sup>57</sup> has a half life of 272 days. What is the average lifetime of an atom of Co<sup>57</sup>?

(7a) Given one atom of Co<sup>57</sup> at noon, what is the probability that it will decay before 1 PM?

(7c) Given 1000 atoms of Co<sup>57</sup> at noon, what is the probability that two or fewer will decay before 1 PM?