M145 Sample Quiz #01a for Jan 7, 2005

Practice problems:

Section 18 (Antiderivatives, p205) 2, 7, 11, 13, 20

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(1-4) Calculate each of the following antiderivatives :

(1) $\int (x^5 + x^2)\, dx$

(2) $\int e^{4x}\, dx$

(3) $\int \sin(2x)\, dx$

(4) $\int \frac{(x^2 + x + 1)\, dx}{x}$

(5) Solve $\frac{dy}{dx} = 2x - 1$, given that $y = 4$ when $x = 2$.

M145 Sample Quiz #01b for Jan 11, 2005

(1-4) Calculate each of the following antiderivatives:

(1) $\int x^{\frac{2}{3}}\, dx$

(2) $\int e^{-3x}\, dx$

(3) $\int (\cos(5x) + \cos(2x))\, dx$

(4) $\int \frac{x^3 + x^2 + x + 1}{\sqrt{x}}\, dx =$

(5) Solve $\frac{dy}{dx} = e^{-2x}$, given that $y(0) = 4$.

M145 Sample Quiz #02 for Tue, Jan 11, 2005

practice problems:
Section 18 (Antiderivatives, p205) 31, 38, 39, 44, 45, 53

Calculate each of the antiderivatives in **(1-4)** .

(1) $\int (x+1)^4 dx$

(2) $\int (3x+2)^5 dx$

(3) $\int 2x(x^2+3)\, dx$

(4) $\int \cos(x)\sin^4(x) dx$

(5) Evaluate $\int_0^1 \frac{x^2}{x^3+1}\, dx$

(6) Evaluate $\int_0^\pi (1+\sin(2x))\, dx$

M145 Sample Quiz #03a for Tue, Jan 18, 2005

(1) $\int (1 + t^4)^8 \cdot 4t^3 dt =$

(2) $\int \frac{\cos(x)}{1 + \sin x} dx =$

(3) Solve $\frac{dy}{dx} = 3x^2 - 2x + 1$, given that $y = 4$ when $x = 2$.

(4) Solve $y'' = 4$, given that $y(0) = 1$ and $y'(0) = 4$.

(5) An object moves along a line, its velocity given by $v(t)$, where $v(t) = 1 + \sin t$. How far has the object gone when $t = \pi$?

(6) Calculate the area in the first quadrant between the curves $y = x^2$ and $x = y^2$. Sketch this region.

M145 Sample #04b For Tue Jan 25, 2005

Practice problems:

Section 19 (The Integral, p216) 14

Section 20 (Fundamental Theorem of Calculus, p225) 20, 21, 22

Section 21 (Areas, p236) 35, 49

(1) An object moves along the x-axis for $t \geq 0$ with acceleration $a(t) = 1 + 3t$. At time $t = 2$, its velocity is 10, and it is 13 units from the roigin. Where is the object when $t = 4$?

(2) Evaluate $\int_0^2 \left(\frac{2x}{x^2+3}\right) dx$ Suggestion: let $u = x^2 + 3$

(3) A population with a limit on its numbers is growing according to the function:

$$N(t) = \frac{100}{1 + 19e^{-0.2\,t}}$$

What is the limit of $N(t)$ as $t \to \infty$?
At what time t does N reach 99% of its limiting value?

(4) Evaluate $\int_0^1 \frac{dt}{e^{-3t}+1}$ Suggestion: First multiply numerator and denominator by e^{3t}

(5) Evaluate $\int_{-\frac{\pi}{2}}^{\pi} (2 + \sin(x))\, dx$

(6) Calculate the area in the first quadrant between the curves $y = x^2$ and $x = y^2$. Sketch this region.

(7) Find the area of the region bounded by the curves $y = x^2$ and $y = (x-2)^2$, and $x = 0$ to $x = 2$. Sketch this region.

M145 Sample #05b For Tue Feb 1, 2005

(1a) Evaluate $\int_2^3 \left(\frac{u}{u-1}\right) du$ Suggestion: let $v = u - 1$;

(1b) Evaluate $\int_4^9 \left(\frac{1}{\sqrt{x}-1}\right) dx$ Suggestion: First let $u = \sqrt{x}$. Then ...

(2) Evaluate $\int_0^2 \left(\frac{2x}{x^2+3}\right) dx$ Suggestion: let $u = x^2 + 3$

(3) Evaluate $\int_0^1 \frac{dt}{e^{-3t}+1}$ Suggestion: First multiply numerator and denominator by e^{3t}. Then ...

(4) Evaluate $\int_{-\frac{\pi}{2}}^{\pi} (2 + \sin(x))\, dx$

(5) Calculate the area in the first quadrant between the curves $y = x^2$ and $x = y^2$. Sketch this region.

(6) Find the area of the region bounded by the curves $y = x^2$ and $y = (x-2)^2$, and $x = 0$ to $x = 2$. Sketch this region.

(7a) Let $f(x) = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2}$ Use the trapezoid rule (2 trapezoids, with $\Delta x = 0.5$) to give an approximate value for the area under the curve from $x = 1$ to $x = 2$.

(7b) Use Simpson's Rule (p 289) with $\Delta x = 0.5$, to give an approximate value for the area under the curve from $x = 1$ to $x = 5$.

M145 Sample Quiz #06b for Tue, Feb 8, 2005

(1) $\int \cot x dx$

(2) $\int_0^{\pi^2} \cos \sqrt{x}\, dx$ suggestion: let $x = u^2$

(3) $\int_0^1 e^{-\sqrt{x}}\, dx$ suggestion: let $x = u^2$

(4) $\int \frac{xdx}{x^4+1}$ suggestion: let $x^2 = u$

(5) $\int \frac{dx}{\sqrt{x^2-16}}$

(6) $\int \frac{dx}{\sqrt{16-x^2}}$

(8) $\int_0^1 \frac{e^t dt}{1+e^t}$

(9) Find inside the circle $x^2 + y^2 = 2$, and to the right of the parabola $y^2 = x$

(10) Let R be the region in the first quadrant bounded by the curve $y = \arcsin x$, the x-axis and the line $x = 1$. Sketch the region R. Express the area of R as a definite integral in **TWO** ways:
(a) with x as the variable of integration; and
(b) with y as the variable of integration. Evaluate both integrals.

M145 Sample Quiz #07b for Tue, Feb 15, 2005

(1) Solve the equation $\frac{dy}{dt} = 2(y-3)$, with $y(0) = 2$.

(2) For each time t (in days), the number of germs G is $y(t)$ (in Billions). The G are growing in such a way that $\frac{dy}{dt} = 2y - 6$ per day.

(2a) Initially (at time $t = 0$) there are 2 Billion germs. What will the numbers be for each time t? Will $y(t)$ ever become 0? If so, when?

(2b) Next suppose that initially there are 5 Billion germs. What will the numbers be for each time t? Will $y(t)$ ever become 0? If so, when?

(3) For each time t (in days), the number of bacteria B is $y(t)$ (in Billions). The bacteria are growing in such a way that $\frac{dy}{dt} = -2y + 6$. Initially (time $t = 0$) there are 2 Billion bacteria. What will its numbers be for each time t? Is there a limit for $y(t)$ as $t \to \infty$?

(4a) Find the antiderivative: $\int \frac{10}{y(10-y)} dy$

(4b) Solve the equation $\frac{dy}{dt} = y(1 - \frac{y}{10})$, with $y(0) = 1$.

(5a) Find the antiderivative: $\int \frac{100}{y(100-y)} dy$

(5b) Solve the equation $\frac{dy}{dt} = 0.5y(100-y)$, with $y(0) = 1$.

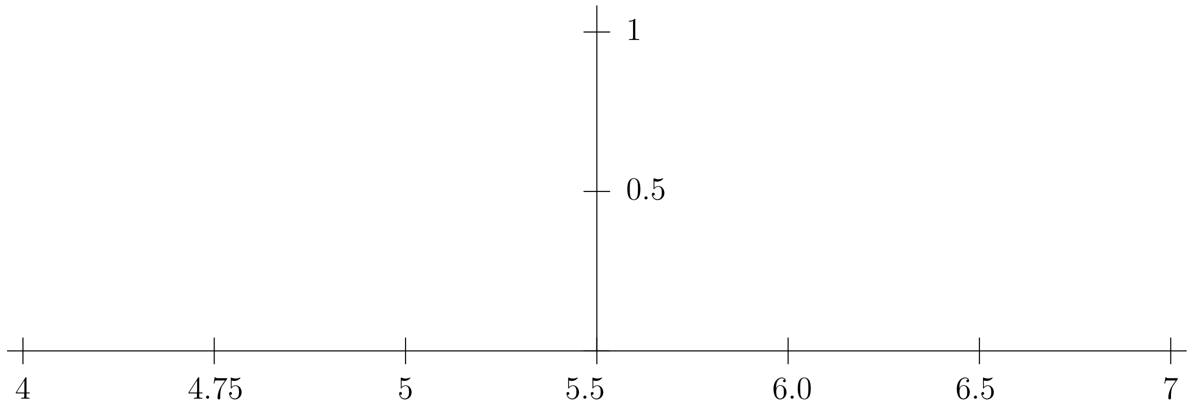
M145 Sample #09b Due Tue March 1, 2005

(1) (Use the table for the Standard Normal Distribution.) The lengths of worms in a certain population is normally distributed, with a mean of 5.5 cm and a standard deviation of 0.5 cm.

(1a) What fraction of the worms have a length of at most 6 cm?

(1b) What fraction of the worms of **(3)** have a length that lies between 6 cm and 6.5 cm?

(1c) Let $F(x)$ be the probability density function for **(1)**. Sketch $F(x)$ and sketch the region which represents the fraction of the individuals of **(1)** that have a length between 6 cm and 6.5 cm.



Place the points corresponding to $F(4)$, $F(4.5)$, $F(5)$, $F(5.5)$, $F(6.0)$, $F(6.5)$, $F(7)$ as accurately as you can.

Note: For the function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, some values are

$f(\pm 3) \approx 0.004$ $f(\pm 2) \approx 0.054$ $f(\pm 1) \approx 0.24$ $f(0) \approx .4$

(2) A pea has a .7 probability of being smooth, and a .3 probability of being wrinkled. If you select 100 peas at random, what is the probability that at least 65 and at most 75 are smooth? (Use the histogram correction)

(3) The length of rattlesnakes is normally distributed, with a mean of 1.1 meters, and a standard deviation of .2 meters. If a large number of rattlesnakes are examined, what fraction of them have a length between 1.0 and 1.3 meters?

(4) The lengths of worms are normally distributed with a mean of 10 cm. 80% of them are shorter than 11.68 cm. What is the probability that the length of a worm is between 8 and 12 cm? Suggestion: First use the table to find the value for z with $\Phi(z) = .80$. This value of z is related to σ by $z \cdot \sigma = 1.68$. Calculate σ. Then use the table again to calculate $P(8 < X < 12)$.

M145 Sample Quiz #10 for Tue, March 8, 2005

Sections	topic	page	Problems
Handout	Poisson Distribution	-	

Use the Poisson approximation for binomial probabilities of rare events.

(1) In the US population, a certain disease occurs in 1.2% of the population. In a sample of 100 people, what is the probability that 0 of them have the disease?

What is the probability that at most 3 of them have the disease?

(2) 100 buses are scheduled to arrive in Seattle during the hour from 1 pm to 2 pm. (Assume each bus arrives at a random time, independent of the other buses.) What is the probability that at most 3 buses arrive during the first minute?

What is the probability that 4 or more buses arrive during the first minute?

(3) A computer chooses 500 random numbers between 0 and 1. What is the probability that 3 or more of them lie between .50 and .51?

(4) A fly trap catches 1000 flies during a 24 hour period. (Assume each fly wanders into the trap at a random time, independent of the other flies.) What is the probability that 4 or more flies are caught during the two minute period between 4:00 and 4:02 pm?

M145 Sample Quiz #11 for Sat, March 12, 2005

(1a) Evaluate $\int_1^2 \left(\frac{x^2}{x+3}\right) dx$ Suggestion: let $u = x + 3$

(1b) Evaluate $\int_0^1 \frac{dt}{e^{-3t}+1}$ Suggestion: First multiply num. and denom. by e^{3t}

(1c) Evaluate $\int_0^{\frac{\pi}{2}} \sin^3\theta \cos\theta d\theta$

(2) Use the trapezoid rule (two trapezoids) to give an approximate value T for $\int_2^4 \ln x\, dx$. Sketch the function $y = \ln x$ for $2 \leq x \leq 4$ and the trapezoids.

(2b) Is T greater than or less than $\int_2^4 \ln x\, dx$? Explain your answer.

(3) In the absence of predators, a population of rabbits grows exponentially and would double in 9 months. A pack of foxes eat rabbits at a rate of 7.7 rabbits per month. Initially there are 150 rabbits. Find the number of rabbits for all times t.

(4a) A certain drug is administered intravenously at the rate of 2 mg per hour. The drug is excreted from the body at the (continuous) rate of 20 % per hour. Initially there is no drug in the body. Find the amount of the drug in the body for all times t.

(4b) How long does it take for the amount of the drug in the body to reach 90 % of its limiting value?

(5a) Find the antiderivative: $\int \frac{10}{y(10-y)} dy$

(5b) Solve the diffenential equation $\frac{dy}{dt} = 0.1y(10-y)$, with $y(0) = 1$.

(6) Assume that the lifetimes of batteries are exponentially distributed; the average life of a battery is 4 years. If you buy a battery today, what is the probability that it will die in the first year? In the first week?

(7) In the US population, a certain disease occurs in 1.5% of the population. In a sample of 100 people, what is the probability that two or more people have the disease?

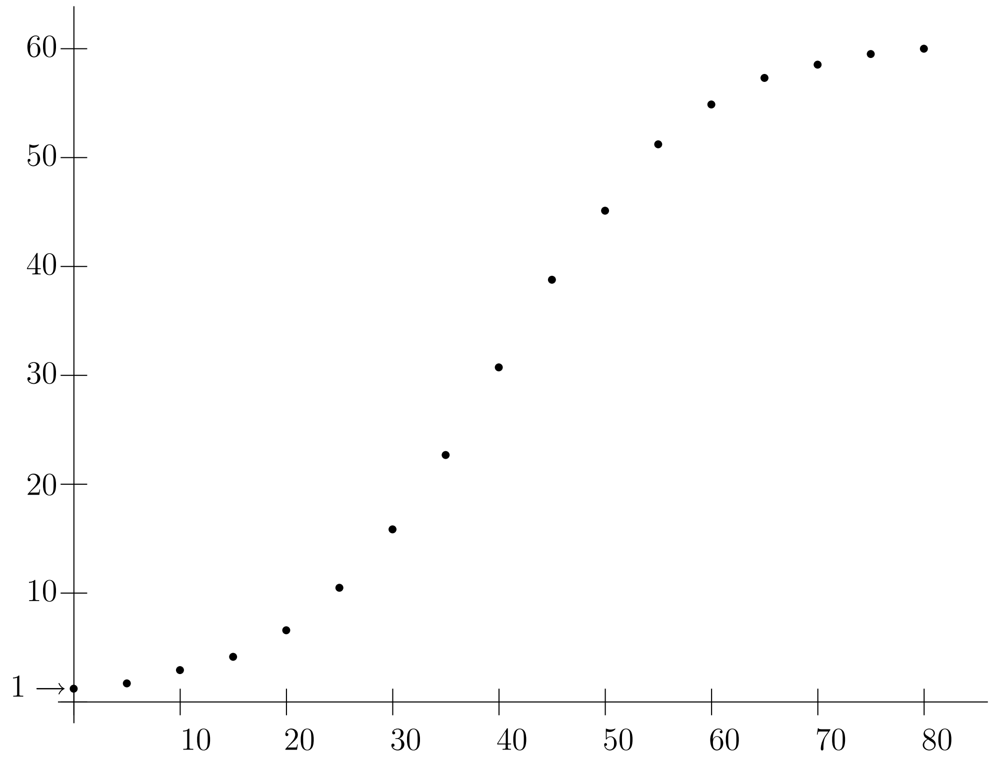
(8) The weights of a certain population are normally distributed. Half of them weigh less than 60 kg, and 75 % weigh less than 70 kg. How many weigh less than 55 kg?

(9) For the offspring of a heterozygous smooth pea plant that is self-pollinated, each plant has a .75 probability of being smooth, and a .25 probability of being wrinkled. If you select 100 plants at random from the offspring, what is the probability that at least 70 and at most 80 are smooth? Use the histogram correction.

(10) Strontium-80 has a half life of 1.77 hours. If you have an atom of Sr-80 at 12 noon, what is the probability that it will decay some time between 1 pm and 2 pm?

(11) The graph shows data for the growth of a population of bacteria which has a limit on its size; $y(t)$ is the amount of bacteria in grams at time t in days; $y(t)$ is assumed to satisfy the logistic equation.

$$y(t) = \frac{K}{1 + ce^{-rt}}$$



Using this data, estimate the values of K, c, and r.

(2) A population with a limit on its numbers is growing according to the function:

$$N(t) = \frac{100}{1 + 19e^{-0.2\,t}}$$

What is the limit of $N(t)$ as $t \to \infty$? When does N reach 99% of its limiting value?