M146 Assignment II Due Thurs, April 6, 2006

Solving Differential Equations

Section 33 (page 368) (Separation of varailbes) 4, 13, 19

Section 34 (page 379) (Logistic Equation) 1-4

Section 37 (page 406) (First Order Linear Equations) 8, 11, 13, 24

Differential equations of the form $y' + ay = f(t)$ are called first order, linear, with constant coefficients. The method of solution is to multiply by e^{at} to obtain

$$e^{at}(y' + ay) = e^{at} f(t)$$

The left-hand side is the derivative of $e^{at}y$, so the equations becomes

$$\left(e^{at}y\right)' = e^{at} f(t) \quad \text{so}$$

$$e^{at}y = \int e^{at} f(t)\, dt \, + \, C$$

(1a) Atoms X radioactively decay to atoms Y, which in turn decay to atoms Z. The amount of X at time (in days) t is $100e^{-2t}$. The half-life of atoms Y is 20 days. Initially there are no atoms of Y or Z.

(1a) What is the rate at which X is decaying to Y?

(1b) What is the intrinsic decay rate for atoms Y?

(1c) Write a differential equation for y', i.e., the rate at which y is changing.

(1d) Solve this equation to calculate the amount of Y for all times t.

M146 Sample Quiz #02 for Thursday, April 6, 2006

(1) Find the solution to $\frac{dy}{dt} = 3(y-1)t^2$, with $y(0) = \frac{1}{2}$.

(2) Solve the differential equation $y' = -2y + 100e^{-4t}$, with $y(0) = 0$.

(3a) A population of rabbits has an instantaneous annual growth rate (birth rate minus natural death rate) of .17 in the absence of predators. Foxes eat rabbits at a rate of 17 rabbits per fox per year. The initial rabbit population is 1750, and there are 20 foxes. Calculate the number of rabbits for all times t. (Assume the number of foxes stays constant.)

(3b) A population of rabbits has an instantaneous annual growth rate (birth rate minus natural death rate) of .17 in the absence of predators. Foxes eat rabbits at a rate of 17 rabbits per fox per year. The initial rabbit population is 1750. How many foxes should you introduce so that the population of rabbits remains constant. (Assume the number of foxes stays constant.)

(4a) The half-life of Sr-90 is 29 years. Suppose Sr-90 is introduced into the environment at the continuous rate of 2 gm per day (730 gm per year). Initially there is no Sr-90. Calculate the amount $y(t)$ of Sr-90 for all times t.

(4b) What is the limiting amount of Sr-90 in the environment?

(5) Radioactive substance X decays to Y with a half-life of 1 day. Y is also radioactive and decays to Z with a half-life of 3 days. Initially there is 100 grams of X and no grams of Y. Find the amount of Y for all times t.