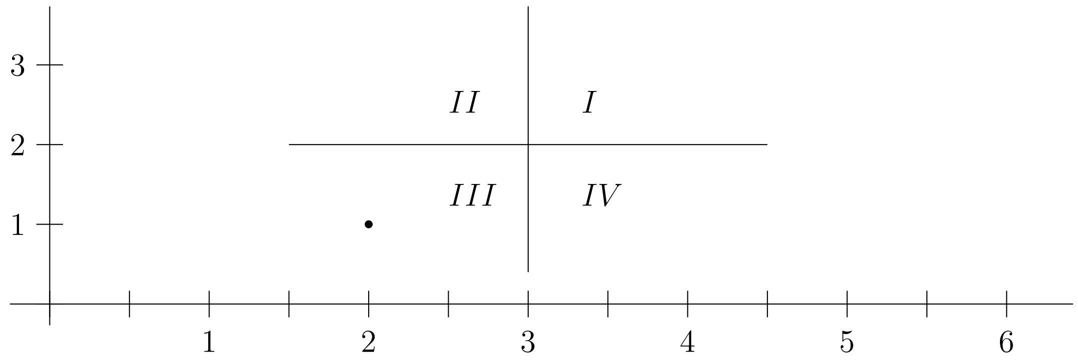
M146 Assignment IX   Due Thurs May 25, 2006

(1) We want an approximate solution to $\begin{matrix} x' = x(2-y) \\ y' = y(-3+x) \end{matrix}$ with $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(1a) Take $\Delta t = 0.5$ and find approximate values for $(x(0.5), y(0.5))$.

(1b) Do two steps, each with $\Delta t = 0.5$, and find approximate values for $(x(1), y(1))$.

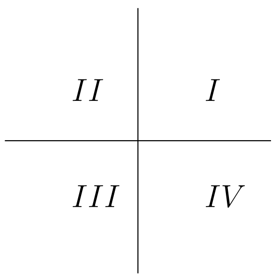
(1c) The lines $x = 3$, and $y = 2$ partition the plane into four regions I, II, III, IV as shown (ignore the $x-$axis and the $y-$axis). The trajectory of **(1b)** starts in region III. What are the next four regions through which the trajectory passes?



(2a) $A = \begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}$ Find the general solution to $x' = Ax$.

(2b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

(2c) The quadrants are numbered as shown below. The trajectory of **(2b)** starts in quadrant IV. In which quadrant (I, II, III or IV) will the trajectory be when $t = 1000$?



(2d) Does $x_2(t)$ ever become 0, and if so, at what time?

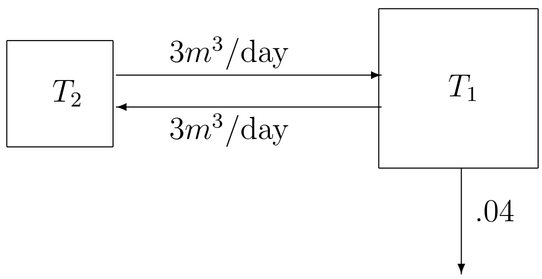
(3) A drug D is taken into the stomach; it then goes into the bloodstream, and is then removed from the bloodstream by the kidneys, and sent to the baldder, where it stays. The half-life of drug D in the stomach is 1 hour; the half-life of drug D in the bloodstream is 8 hours. 100 mg of drug D are taken at time $t = 0$. Let $x(t)$ be the amount of D in the stomach at time t, and let $y(t)$ be the amount of D in the blood at time t. let $z(t)$ be the amount of D in the bladder at time t. Draw a compartment diagram for x, y and z.

(3a) Write a system of differential equations for $x(t)$ and $y(t)$.

(3b) Solve this system for $(x(t), y(t))$ for all t.

(3c) What is the maximum amount of drug D that is ever present in the blood, and at what time does it occur?

(4) Two tanks T_1 and T_2 are connected as in the following diagram. Tank 1 contains 100 m^3 of liquid, tank 2 contains 25 m^3 of liquid. The hoses connecting the two tanks carry 3 m^3 of liquid per day each way.



There is some bacteria in the two tanks. Tank 1 is exposed to ultraviolet radiation that kills the bacteria at the continuous rate of 4% of the bacteria per day. Let $x_1(t)$ be the quantity of bacteria in tank 1, and $x_2(t)$ be the quantity of bacteria in tank 2. Write a system of differential equations for the quantities x_1 and x_2.

At $t = 0$, tank 1 contains 13 units of bacteria, and tank 2 contains 0 unit of bacteria. Solve these equations for the amounts of bacteria in each tank for all t. What is the total amount of bacteria remaining in the two tanks after 30 days?