

M146 Assignment X Due Friday, June 2, 2006

(1a) Verify $\frac{1+2i}{3-4i} + \frac{2-i}{5i} = -\frac{2}{5}$

(1b) Verify $\frac{5}{(1-i)(2-i)(3-i)} = \frac{1}{2}i$

(1c) Verify $(1-i)^4 = -4$

(2) Show that $z + \bar{z} = 2 \cdot \text{Re}z$.

(3) Let $z_1 = 4 + 3i$ and $z_2 = 5 + 12i$. Calculate $|z_1 z_2|$ and $|z_1||z_2|$.

(4) Find all complex numbers $z = x + iy$ such that $z^2 = 1 + i$.

(5) Let $z_1 = 3i$ and $z_2 = 2 - 2i$

(5a) Plot the points $z_1, z_2, z_1 + z_2, z_1 - z_2$ and \bar{z}_2 .

(5b) Compute $|z_1 + z_2|$ and $|z_1 - z_2|$.

(5c) Express z_1 and z_2 in polar form.

(6) Show that $e^{i\theta_1} e^{i\theta_2} = e^{i(\theta_1 + \theta_2)}$.

(Use Euler's Formula for $e^{i\theta_1}, e^{i\theta_2}$ and the addition formulas for cos and sin.)

(7) Use (6) to show that $(e^{i\theta})^{-1} = e^{-i\theta}$, that is, Show that $e^{-i\theta} e^{i\theta} = 1$.

(8) Let $z_1 = 6e^{i\pi/3}$ and $z_2 = 2e^{-i\pi/6}$. Plot $z_1, z_2, z_1 z_2$ and $\frac{z_1}{z_2}$.

(9) Find all complex numbers z which satisfy $z^3 = -1$.

(10) Find all complex numbers $z = re^{i\theta}$ which satisfy $z^2 = \sqrt{2}e^{i\pi/4}$.