

## First Order Linear Differential Equations

Differential equations of the form  $y' + ay = f(t)$  are called first order, linear, with constant coefficients. The method of solution is to multiply by  $e^{at}$  to obtain

$$e^{at}(y' + ay) = e^{at}f(t)$$

The left-hand side is the derivative of  $e^{at}y$ , so the equations becomes

$$\begin{aligned}(e^{at}y)' &= e^{at}f(t) \\ e^{at}y &= \int e^{at}f(t) dt + C \\ y &= \dots\end{aligned}$$

**Example** Solve the differential equation  $y' = -3y + e^{-4t}$ , with  $y(0) = 3$ .

$$\begin{aligned}y' + 3y &= e^{-4t} \\ e^{3t}(y' + 3y) &= e^{-t} \\ (e^{3t} \cdot y)' &= e^{-t} \\ e^{3t} \cdot y &= -e^{-t} + C \\ y &= -e^{-4t} + Ce^{-3t} \\ 3 &= -1 + C; \quad 4 = C \\ y &= -e^{-4t} + 4e^{-3t}\end{aligned}$$