M146 Quiz I April 1, 2005 (50 pts) Name: ....

Problem **(1)** on this page, problem **(1)** on reverse page.

(1) Solve $\dfrac{dy}{dt} = -2\,t\,(y-2)$ with $y(0) = 5\,.$

(2) A body is infected by bacteria. When first observed it is estimated that there are 10 million bacteria, and 3 days later there are 15 million bacteria. At three days, a drug is administered which kills the bacteria at the rate of 3 million per day. Find the number of bacteria present at each time t. When are all the bacteria dead?

M146 Test II April 7, 2005 Name: ....

(1) Find the steady state values for the following equation, and for each steady state value determine whether it is stable, or unstable.

$$\frac{dy}{dt} = 2\,y(2-y)(10-y)$$

(1b) If $f(0) = 1$, what is $\lim_{t\to\infty} f(t)$?

(1c) If $f(0) = 5$, what is $\lim_{t\to\infty} f(t)$?

(1d) If $f(0) = 50$, what is $\lim_{t\to\infty} f(t)$?

(2) A yeast is in the form of a disc of constant uniform thickness. The yeast is growing exponentially, and its area doubles every 3 hours. How fast is the radius r increasing when $r = 5$ cm?

(3) Solve: $\dfrac{dy}{dt} = 3t^2y^2,$ $y(0) = 1\,.$ Obtain a formula for y as a function of t.

(4) Solve: $y' = \dfrac{1}{2}y(1-y)$ with $y(0) = 0.1$. Note: $\dfrac{1}{y(1-y)} = \dfrac{1}{y} + \dfrac{1}{1-y}$.
Obtain a formula for y as a function of t.

M146 Test III April 14, 2005 Name: ....

(1) Solve the differential equation $y' = -y + 10e^{-2t} + 1$ with $y(0) = 2$.

(2) A substance S is taken into the stomach, is absorbed into the blood, and then goes from the blood to the liver. S goes from the stomach to the blood at the (countinous) rate of 40 % per hour. S is removed from the blood at the continuous rate of 10 % per hour. In addition some more substance S is injected directly into the bloodstream at the constant rate of 3 mg per hour. Initially 100 mg of the S are present in the stomach, and 20mg in the blood. Let $y(t)$ be the amount of S in the blood at time t.

(2a) Write a differential equation for the rate of change of y.

(2b) Solve this differential equation to find $y(t)$, the amount of S in the blood at time t.

(3) A drug D is ingested, goes into the blood, and is excreted. The concentration of drug D in the blood is given by the formula: $C(t) = 10e^{-2t} - 10e^{-10t}$. What is the maximum concentration and at what time does it ocur?

(4) Radioactive substance X decays to Y with a half-life of 1 day. Y is also radioactive and decays to Z with a half-life of 3 days. Initially there is 100 grams of X and 20 grams of Y. Find the amount of Y for all times t.

M146 Test IV April 21, 2005 Name: ....

(1) Find all solutions of $\begin{array}{rcrcr} 2x_1 & + & 4x_2 & = & 2 \\ 3x_1 & + & 7x_2 & = & 8 \end{array}$

(2) $A = \begin{bmatrix} -1 & -2 & 3 \\ 2 & 3 & -2 \\ 1 & 1 & 1 \end{bmatrix} \quad c = \begin{bmatrix} -5 \\ 4 \\ 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}$

(2a) Calculate $A\,c$

(2b) Calculate $A\,B$

(3) Find all solutions of $\begin{array}{rcrcrcr} 3x_1 & + & 4x_2 & + & 2x_3 & = & 2 \\ 6x_1 & + & 9x_2 & + & 5x_3 & = & 6 \end{array}$

(4) $A = \begin{bmatrix} 1 & 0 & 0 & 2 & 3 \\ 1 & 1 & 0 & 3 & 7 \\ 1 & 1 & 1 & 3 & 2 \\ 1 & 0 & 1 & 2 & -2 \end{bmatrix}$ Reduce A to Row Reduced Echelon Form.

(4b) What is the rank of A? What is the nullity of A?

M146 Test V April 28, 2005 Name: ....

(1) Find all solutions of
$$\begin{array}{ccccc} y & + & x & = & 3 \\ z & - & y & = & -1 \\ x & + & z & = & 2 \end{array}$$

(2) $A = \begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ -1 & 0 & 1 & -2 & 1 \\ 1 & 0 & 1 & 0 & 5 \\ 2 & 0 & 1 & 1 & 7 \end{bmatrix}$

What is the rank of A? What is the nullity of A? Give all solutions to $Ax = 0$.

(3) A population is divided into calves, yearlings, adults. The Leslie matrix for is

$L = \begin{bmatrix} 0 & 0 & 3.2 \\ 0.5 & 0 & 0 \\ 0 & 0.4 & 0.5 \end{bmatrix}$. The inutial population distribution is $\begin{bmatrix} 300 \\ 150 \\ 100 \end{bmatrix}$.

What will be the population distribution after 1 year? After 2 years?

(4) A population is divided into newborns and adults. Each adult produces 3 newborns per year. Each year 10 % of the newborns survive to be adults and 70 % of the adults survive to the next year. Find a population $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ which is stable, and which totals 1200.

M146 Worksheet 05 May 2, 2005 Name: ....

(1) $A = \begin{bmatrix} 0 & 0 & 1 & 2 & 3 \\ 1 & 2 & 0 & 1 & 2 \\ 1 & 2 & 1 & 3 & 5 \\ 1 & 2 & -1 & -1 & -1 \end{bmatrix}$

What is the rank of A? What is the nullity of A? Give all solutions to $Ax = 0$.

(2) A population is divided into calves, and adults. The Leslie matrix for this population is: $L = \begin{bmatrix} 0 & 2.4 \\ 0.2 & 0.8 \end{bmatrix}$. One eigenvalue for L is $\lambda = 1.2$. What is the stable population frequency distribution?

M146 Test VI May 5, 2005 Name: ....

(1) $A = \begin{bmatrix} 0 & 0 & 1 & 2 & 3 \\ 1 & 2 & 0 & 1 & 2 \\ 1 & 2 & 1 & 3 & 5 \\ 1 & 2 & -1 & -1 & -1 \end{bmatrix}$ $b = \begin{bmatrix} 5 \\ 4 \\ 9 \\ -1 \end{bmatrix}$. Give all solutions to $Ax = b$.

(2) A population is divided into newborns and adults. Each adult produces 3.3 newborns per year. Each year 10 % of the newborns survive to be adults and 80 % of the adults survive to the next year. (The largest eigenvalue for L is 1.1). What is the stable population frequency distribution?

(3) A population is divided into calves, yearlings, adults. Each adult produces 3 calves per year; 40 % of the calves survive to be yearlings; 50 % of the yearlings survive to become adults; 40 % of the adults survive another year. Let L be th Leslie matrix for this population. The largest eigenvalue for L is $\lambda = 1.0$. What are the numbers of calves, yearlings and adults in a stable population which has a total of 2600 animals?

(1) Find all solutions of $\begin{array}{rcrcr} y & + & x & = & 3 \\ z & - & y & = & -1 \\ x & + & z & = & 2 \end{array}$

(2) $A = \begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ -1 & 0 & 1 & -2 & 1 \\ 1 & 0 & 1 & 0 & 5 \\ 2 & 0 & 1 & 1 & 7 \end{bmatrix}$

What is the rank of A? What is the nullity of A? Give all solutions to $Ax = 0$.

(3) A population is divided into calves, yearlings, adults. The Leslie matrix for is $L = \begin{bmatrix} 0 & 0 & 3.2 \\ 0.5 & 0 & 0 \\ 0 & 0.4 & 0.5 \end{bmatrix}$. The inutial population distribution is $\begin{bmatrix} 300 \\ 150 \\ 100 \end{bmatrix}$.

What will be the population distribution after 1 year? After 2 years?

(4) A population is divided into newborns and adults. Each adult produces 3.3 newborns per year. Each year 30 % of the newborns survive to be adults and 20 % of the adults survive to the next year. What is the stable population frequency distribution?

M146 Test VIa May 6, 2005 Name: ....

(1) $A = \begin{bmatrix} 1 & 3 & 0 & 4 \\ 1 & 3 & 1 & 6 \\ -1 & -3 & 1 & -2 \end{bmatrix}$ $b = \begin{bmatrix} 5 \\ 11 \\ 1 \end{bmatrix}$. Give all solutions to $Ax = b$.

(2) A population is divided into newborns and adults. Each adult produces 3.3 newborns per year. Each year 10 % of the newborns survive to be adults and 80 % of the adults survive to the next year. (The largest eigenvalue for L is 1.1). What is the stable population frequency distribution?

(3) A population is divided into calves, yearlings, adults. Each adult produces 3 calves per year; 40 % of the calves survive to be yearlings; 50 % of the yearlings survive to become adults; 40 % of the adults survive another year. Let L be th Leslie matrix for this population. The largest eigenvalue for L is $\lambda = 1.0$. What are the numbers of calves, yearlings and adults in a stable population which has a total of 2600 animals?

M146 Worksheet 06 May 6, 2005

(1) Find all solutions of $\begin{array}{ccccc} y & + & x & = & 3 \\ z & - & y & = & -1 \\ x & + & z & = & 2 \end{array}$

(2) $A = \begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ -1 & 0 & 1 & -2 & 1 \\ 1 & 0 & 1 & 0 & 5 \\ 2 & 0 & 1 & 1 & 7 \end{bmatrix}$

What is the rank of A? What is the nullity of A? Give all solutions to $Ax = 0$.

(3) A population is divided into calves, yearlings, adults. The Leslie matrix for is

$L = \begin{bmatrix} 0 & 0 & 3.2 \\ 0.5 & 0 & 0 \\ 0 & 0.4 & 0.5 \end{bmatrix}$. The inutial population distribution is $\begin{bmatrix} 300 \\ 150 \\ 100 \end{bmatrix}$.

What will be the population distribution after 1 year? After 2 years?

(4) A population is divided into newborns and adults. Each adult produces 3.3 newborns per year. Each year 30 % of the newborns survive to be adults and 20 % of the adults survive to the next year. What is the stable population frequency distribution?

M146 Quiz VII Thursday, May 12, 2005 Name: ....

(1) $A = (4, 1, 1)$ and $B = (2, 4, 7)$. and $C = (5, -1, 3)$.

(1a) Find the angle between $\vec{AB}$ and $\vec{AC}$.

(1b) Give the equation for the line through A and B in parametric form.

(2) $v_1 = (1, 3)$ $v_2 = (2, 9)$, and $w = (1, 9)$. Find constants c_1 and c_2 so that $w = c_1v_1 + c_2v_2$.

(3) Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. The solutions to $Ax = b$ can be described as the points on a line L. Give the equation for L is parametric form.

(4) $A = \begin{bmatrix} 2 & 3 \\ -1 & 6 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

M146 Quiz VIII Thursday, May 19, 2005 Name: ....

(1) There is SO_2 in the Air, Water, and Plants. From measurements, and find that
20 % of the SO_2 in the Air is transferred to the Water;
10 % of the SO_2 in the Air is transferred to the Plants;

30 % of the SO_2 in the Water is transferred to the Air;
10 % of the SO_2 in the Water is transferred to the Plants;

30 % of the SO_2 in the Plants is transferred to the Air;
20 % of the SO_2 in the Plants is transferred to the Water;

(1a) What is the transfer matrix for this model?

(1b) Find the stationary state distribution of 1200 lbs of SO_2.

(2a) $A = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}$. Find eigenvalues λ_1, λ_2 and the corresponding eigenvectors v_1, v_2 for A. (Both eigenvalues are integers.)

(3) Sr-90 is deposited is cycled through the ecosystem, We make four compartments: G = (Grasses), S = (Soil), O = (Organic; dead organic matter), and W = (Water). The time-step is one month, and (some of) the transfer matrix coefficients are found experimentally to be:

$$T = \begin{bmatrix} - & .01 & 0 & 0 \\ .05 & - & .2 & 0 \\ .1 & 0 & - & 0 \\ 0 & .01 & 0 & - \end{bmatrix}$$

Initially there are 20 tons, 60 tons, 15 tons and 20 tons of Sr-90 in G, S, O, and W respectively.

(3a) What will be the distribution of the Sr-90 after one month?

(3b) What will be the limiting distribution of the Sr-90?

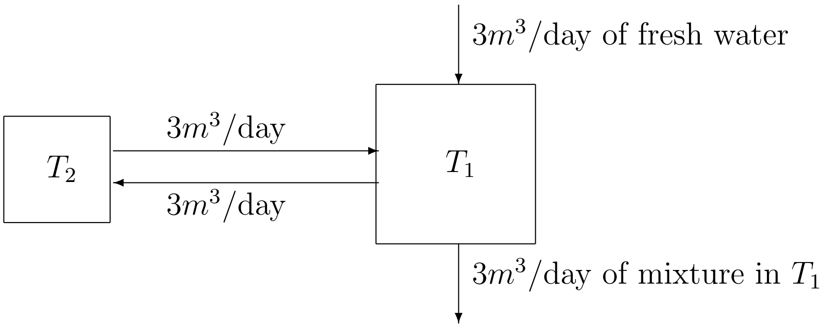
M146 Quiz IX Thursday, May 26, 2005 Name: ....

(1) Let $A = \begin{bmatrix} -3 & 2 \\ 2 & -6 \end{bmatrix}$

(1a) Find the solution to the system $x' = AX$ for which $x_1(0) = 0, \quad x_2(0) = 100.$

(1b) What are the maximum values of $x_1(t)$ and $x_2(t)$ for $t \geq 0$?

(2) Two tanks T_1 and T_2, each with a salt water mixture, are connected as in the following diagram. Tank 1 contains 15 m^3; Tank 2 contains 10 m^3. The hoses connecting the two tanks carry 3 m^3 of liquid per day each way, as indicated..



An outlet pipe takes the mixture in Tank 1 out at $3m^3$/day; an inlet pipe brings fresh water into Tank 1 at $3m^3$/day, as indicated. Let $x_1(t)$ be the grams of salt in Tank 1, and $x_2(t)$ be the grams of salt in Tank 2. Write a system of differential equations for the quantities x_1 and x_2. Suppose that at $t = 0$, Tank 2 contains 100 grams of salt, and Tank 1 contains 0 grams of salt. Solve these equations for the amounts of salt in each tank for all $t \geq 0$.

M146 Assignment IX Due Thurs May 26, 2005

Section 56 (page 639) 13, 14

(1) Let $A = \begin{bmatrix} -3 & 0 \\ 4 & 2 \end{bmatrix}$

(1b) Find the eigenvalues and eigenvectors for the matrix A.

(1b) Find the general solution to the system $x' = AX$.

(1c) Find the solution to the system for which $x_1(0) = -5, \quad x_2(0) = -5$.

(2) $A = \begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(2b) Find the general solution to the system $x' = AX$.

(2c) Find the solution to the system for which $x_1(0) = 6, \quad x_2(0) = 14$.

(3) $A = \begin{bmatrix} -.3 & .2 \\ .1 & -.2 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(3b) Find the general solution to the system $x' = AX$.

(3c) Find the solution to the system for which $x_1(0) = 10, \quad x_2(0) = 0$.

M146 Sample Quiz #09 for Thursday, May 26, 2005

(1) Let $A = \begin{bmatrix} -3 & 0 \\ 4 & 2 \end{bmatrix}$

(1b) Find the eigenvalues and eigenvectors for the matrix A.

(1b) Find the general solution to the system $x' = AX$.

(1c) Find the solution to the system for which $x_1(0) = -5, \quad x_2(0) = -5$.

(2) $A = \begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(2b) Find the general solution to the system $x' = AX$.

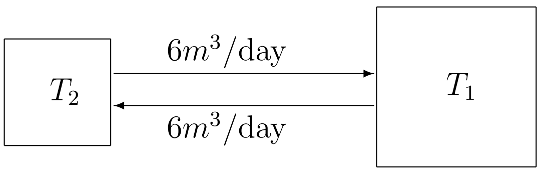
(2c) Find the solution to the system for which $x_1(0) = 6, \quad x_2(0) = 14$.

(3) $A = \begin{bmatrix} -.3 & .2 \\ .1 & -.2 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(3b) Find the general solution to the system $x' = AX$.

(3c) Find the solution to the system for which $x_1(0) = 10, \quad x_2(0) = 0$.

(4) Two tanks T_1 and T_2 are connected as in the following diagram. Tank 1 contains 60 m^3 of liquid, tank 2 contains 20 m^3 of liquid. The hoses connecting the two tanks carry 6 m^3 of liquid per day each way. A substance S is disolved in the liquid, and is assumed to be thououghly mixed.



Let $x_1(t)$ be the quantity of substance S in tank 1; $x_2(t)$ be the quantity of S in tank 2. Write a system of differential equations for the quantities x_1 and x_2.

Suppose that at $t = 0$, Tank 2 contains 120 gms of substance S, and Tank 1 contains 0 gms of S. Solve these equations for the amounts of S in each tank for all t.

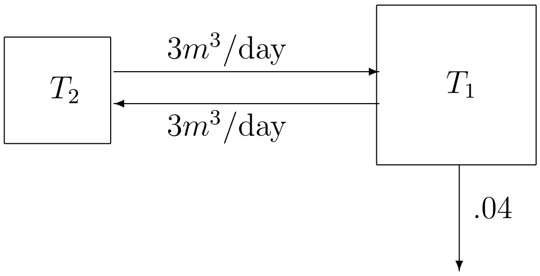
M146 Assignment X Due Thurs June 2, 2005

(1) $A = \begin{bmatrix} -.07 & .12 \\ .03 & -.12 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(1b) Find the general solution to the system $x' = AX$.

(1c) Find the solution to the system for which $x_1(0) = 13$, $x_2(0) = 0$.

(2) Two tanks T_1 and T_2 are connected as in the following diagram. Tank 1 contains 100 m^3 of liquid, tank 2 contains 25 m^3 of liquid. The hoses connecting the two tanks carry 3 m^3 of liquid per day each way.



There is some bacteria in the two tanks. Tank 1 is exposed to ultraviolet radiation that kills the bacteria at the continuous rate of 4% of the bacteria per day. Let $x_1(t)$ be the quantity of bacteria in tank 1, and $x_2(t)$ be the quantity of bacteria in tank 2. Write a system of differential equations for the quantities x_1 and x_2. ß

Suppose that at $t = 0$, tank 1 contains 13 units of bacteria, and tank 2 contains 0 unit of bacteria. Solve these equations for the amounts of bacteria in each tank for all t. What is the total amount of bacteria remaining in the two tanks after 30 days?

(3) A drug D is taken into the stomach; it then goes into the bloodstream, and is then removed from the bloodstream by the kidneys, and sent to the baldder, where it stays. The drug D goes from the stomach to the blood at the (instantaneous) rate of 50 % per hour. The drug D goes from the blood to the bladder at the (instantaneous) rate of 10 % per hour. 100 mg of drug D are taken at time $t = 0$. Let $x(t)$ be the amount of D in the stomach at time t, and let $y(t)$ be the amount of D in the blood at time t. let $z(t)$ be the amount of D in the bladder at time t. Draw a compartment diagram for x, y and z. Write a system of differential equations for $x(t)$ and $y(t)$. Solve this system for $(x(t), y(t))$ for all t. What is the maximum amount of drug D that is ever present in the blood, and at what time does it occur?

M146 Final June 4, 2005 at 1:30 in Gugg 317 Name: ....

(1) A population (females only) is divided into calves, yearlings, and adults.

Each adult gives birth to 2.4 calves per year.

Each year 60 % of the calves survive to become yearlings.

Each year 50 % of the yearlings survive to become adults.

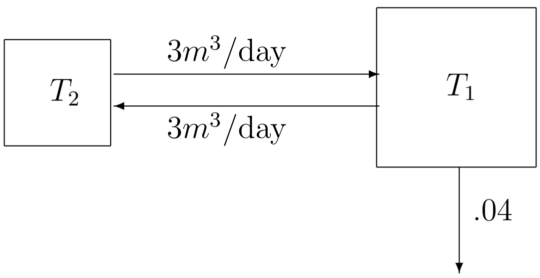
Each year 70 % of the adults survive.

(1a) Write the Leslie matrix L for this population.

(1b) The largest eigenvalue for L is $\lambda = 1.2$. What are the percentages of calves, yearlings and adults in the stable population distibution?

(2) Let $A = \begin{bmatrix} 1 & 1 & 2 & 1 & 2 \\ 2 & 1 & 5 & 2 & 4 \\ 3 & 2 & 7 & 3 & 6 \\ 4 & 3 & 9 & 5 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} 9 \\ 15 \\ 24 \\ 37 \end{bmatrix}$. Find all solutions of $Ax = b$.

(3) Two tanks T_1 and T_2 are connected as in the following diagram. Tank 1 contains 100 m^3 of liquid, tank 2 contains 25 m^3 of liquid. The hoses connecting the two tanks carry 3 m^3 of liquid per day each way.



There is some bacteria in the tanks. Tank 1 is exposed to ultraviolet radiation that kills the bacteria at the continuous rate of 4% of the bacteria per day. Let $x_1(t)$ be the quantity of bacteria in tank 1, and $x_2(t)$ be the quantity of bacteria in tank 2. Initially, tank 1 contains 1300 grams of bacteria, and tank 2 contains 0 unit of bacteria. Calculate the amounts of bacteria in each tank for all t.

(4) A drug D is taken into the stomach; it then goes into the bloodstream, and is removed from the bloodstream, and goes into the baldder, where it stays. The half-life of drug D in the stomach is 1 hour; the half-life of drug D in the bloodstream is 7 hours. 1200 mg of drug D are taken at time $t = 0$. Let $x(t)$ be the amount of D in the stomach at time t, and let $y(t)$ be the amount of D in the blood at time t. let $z(t)$ be the amount of D in the bladder at time t.

(4a) Write a system of differential equations for $x(t)$ and $y(t)$. and solve this system for $(x(t), y(t))$ for all t.

(4b) What is the maximum amount of drug D that is ever present in the blood, and at what time does it occur?