M146 Quiz #01 03/30/2006 Name: ....

(1a) Find the steady state values for the differential equation $y' = 2\,y(y-3)(100-y)$. For each steady state value determine whether it is stable, or unstable..

(1b) Suppose $y = f(t)$ solves **(1a)** with $f(0) = 1$. What is $\lim_{t\to\infty} f(t)$?

(1c) Suppose $y = g(t)$ solves **(1a)** with $g(0) = 4$. What is $\lim_{t\to\infty} g(t)$?

(2a) Find the solution $\frac{dy}{dt} = -2y + 20\,, \qquad y(0) = 3\,.$

(2b) At what time does y reach 90 % of its steady state value?

(3a) Suppose $y = f(t)$ is the solution to $\frac{dy}{dt} = 0.1y(10-y)$, with $y(0) = 1$. Without solving the equation, what is $\lim_{t\to\infty} f(t)$?

(3b) Solve the differential equation $\frac{dy}{dt} = 0.1y(10-y)$, with $y(0) = 1$.

(3c) At what time does y reach 50 % of its steady state value?

M146 Test #02 04/06/2006 Name: ....

(1) Find the solution to $\dfrac{dy}{dt} = 2(y-1)t$ with $y(0) = 3$.

(2a) Drug D is administered intravenously at the rate of 6.93 mg per hour. The amount of D in the body decreases exponentially; the half-life of D in the body is 3 hours. Initially there is no D in the body. Calculate the amount of D in the body for all times t.

(2b) At what time does the amount of D in the body to reach 90 % of its limiting value?

(3) Substance S is taken into the stomach, is absorbed by the blood, then removed from the blood by the kidney. The half-life of S in the stomach is 6 hours. The half-life of S in the blood is 2 hours. Initially 100 mg of the S are in the stomach, and none in the blood.

(3a) Write a differential equation for the rate of change of y. Then solve this to find the amount of S in the blood for all times t.

(3b) At what time does the amount of S in the blood reach its maximum?

M146 Test #03 04/13/2006 Name: ....

(1) A substance S is taken into the stomach, is absorbed into the blood, then is removed from the blood by the liver. S goes from the stomach to the blood at the (countinous) rate of 30 % per hour. S is removed from the blood at the (continuous) rate of 20 % per hour. In addition, more S is injected directly into the bloodstream at the constant rate of 10 mg per hour. Initially 100 mg of S is present in the stomach, and none in the blood. Let $y(t)$ be the amount of S in the blood at time t.

(1a) Write a differential equation for the rate of change of y.

(1b) Solve **(1a)** to find the amount of S in the blood for all times t.

(1c) What is the limiting amount of S in the blood?

(2) $A = \begin{bmatrix} 2 & 4 \\ 3 & 7 \end{bmatrix}$ and $b = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$. Find all solutions of $Ax = b$.

(3) $A = \begin{bmatrix} 2 & 4 \\ 3 & 7 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Find all solutions of $Ax = b$.

(4) Find all solutions of

$$\begin{array}{rcrcrcl} x_1 & + & x_2 & + & 3x_3 & = & 2 \\ 2x_1 & + & 5x_2 & + & 12x_3 & = & 7 \\ 3x_1 & + & 6x_2 & + & 15x_3 & = & 9 \end{array}$$

M146 Test #04 April 20, 2006

(1) Let $A = \begin{bmatrix} 1 & -1 & 4 \\ -2 & 1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$. Find all solutions to $Ax = b$.

(2a) A population is divided into newborns and adults. Each adult produces 2 newborns per year; 30 % of the newborns survive to become adults; each year 70 % of the adults survive to the next year. Give the Leslie matrix L for this population.

(2b) One eigenvalue (the largest) for L is $\lambda = 1.2$. Find a population distribution of newborns and adults which is unchanged in each generation.

That is, find $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ with $x_1 + x_2 = 1$ and $Lx = 1.2x$.

(3a) A population is divided into calves, yearlings, adults. The Leslie matrix for this population is $L = \begin{bmatrix} 0 & 0 & 2 \\ 0.3 & 0 & 0 \\ 0 & 0.5 & 0.7 \end{bmatrix}$. Give the meaning of each non-zero number in this matrix.

(3b) One eigenvalue (the largest) for L is $\lambda = 1.0$. Find a stable population which totals 360. That is find $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ with $x_1 + x_2 + x_3 = 360$ and $Lx = x$.

M146 Quiz #05 for Thursday, April 27, 2006

(1) $A = \begin{bmatrix} 1 & 1 & -6 & 1 & -1 \\ 1 & -2 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 & 2 \\ 2 & 0 & -8 & 4 & 2 \end{bmatrix}$ $\quad \vec{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

What is the rank of A? What is the nullity of A? Give all solutions to $Ax = \vec{0}$.

(2) Let $A = \begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ -9 \end{bmatrix}$, $\vec{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Find all solutions of $Ax = \vec{0}$. Find all solutions of $Ax = b$. The solutions to $Ax = b$ can be described as the points on a line L. Give the equation for L is parametric form.

(3) $A = \begin{bmatrix} 1 & 6 \\ -2 & 9 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(4) Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 7 & -3 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Find all solutions of $Ax = 0$. Find all solutions of $Ax = b$. The solutions to $Ax = b$ can be described as the points on a line L. Give the equation for L is parametric form.

(5a) $A = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and $b = \begin{bmatrix} 7 \\ 4 \\ 8 \end{bmatrix}$. Use the method of overdetermined systems to find the value of x that minimizes the length of the vector $Ax - b$.

M146 Test 06 Thursday, May 04, 2006

(1) . Find the point on the line $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \cdot t$ closest to the point $b = \begin{bmatrix} 5 \\ 5 \\ 7 \end{bmatrix}$.

(2) Find the best least squares solution to:

$$\begin{array}{rcrcl} -x_1 & + & 2x_2 & = & 6 \\ & & -2x_2 & = & 3 \\ 2x_1 & - & 6x_2 & = & 3 \end{array}$$

(3) The following data were obtained for the length y in centimeters of a salamander versus the age t in weeks. Find a linear function $y = b + mt$ which best fits the data.

Age t	1	3	4
Length y	3	7	9

(4) A colony of yeast Y is decaying exponentially. The data shows the amount (y in grams) of Y at times (t in days). Find an exponential function $y = Ae^{rt}$ which best fits the data.

t	1	5	10
y	12	27	74

M146 Test 07 Thursday, May 11, 2006 Name: ....

(1) $T = \begin{bmatrix} .6 & 0 & .4 \\ .3 & 1 & .1 \\ .1 & 0 & .4 \end{bmatrix}$ is the transfer matrix for a three compartment model.

(1a) Make a 3 compartment diagram with arrows showing the transfer.

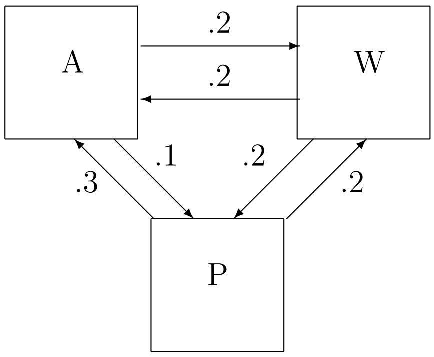
(1b) Initially there are 500 tons of substance S in compartment #1, 300 tons in # 2, and 200 tons in # 3. What is the stable distribution? Explain your answer. In particular how you know the stable distribution with (almost) no calculations.

(2) $T = \begin{bmatrix} .6 & .2 & .2 \\ 0 & .7 & 0 \\ .4 & .1 & .8 \end{bmatrix}$ is the transfer matrix for a three compartment model.

Initially there are 1200 tons of substance S in compartment #1. Find the stable distribution.

(3) T is the 5 by 5 matrix for a 5 compartment model. What is the interpretation of the number $T_{2,3}$.

(4) The diagram shows the transfer per year of S between 3 compartments A, W, and P.



(4a) Give the transfer matrix for this compartment model.

(4b) Initially there are 300 tons of substance S in compartment A, 100 in W, and 500 in P. Find the steady state distribution of S.

M146 Quiz 08 Thurs May 18, 2006 Name: ....

(1) Let $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$, kip1cm $x = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

(1a) Find the general solution to the system $x' = Ax$.

(1b) Find the solution for which $x_1(0) = 5, \quad x_2(0) = 2$.

(2) Let P be the papallelogram in the plane with vertices $(0,0)$, $(2,1)$, $(3,4)$ and $(1,3)$. Calculate the area of P.

(3) Calculate $\det \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ 2 & 6 & 11 \end{bmatrix}$

(4a) $A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$ The three eigenvalues are $\lambda_1 = 1$, $\lambda_2 = -2$, $\lambda_3 = 3$.

(4a) Find the general solution to the system $x' = Ax$, where $x = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$

(4b) Find the solution for which $x_1(0) = 9, \quad x_2(0) = 3. \; x_3(0) = 1$.

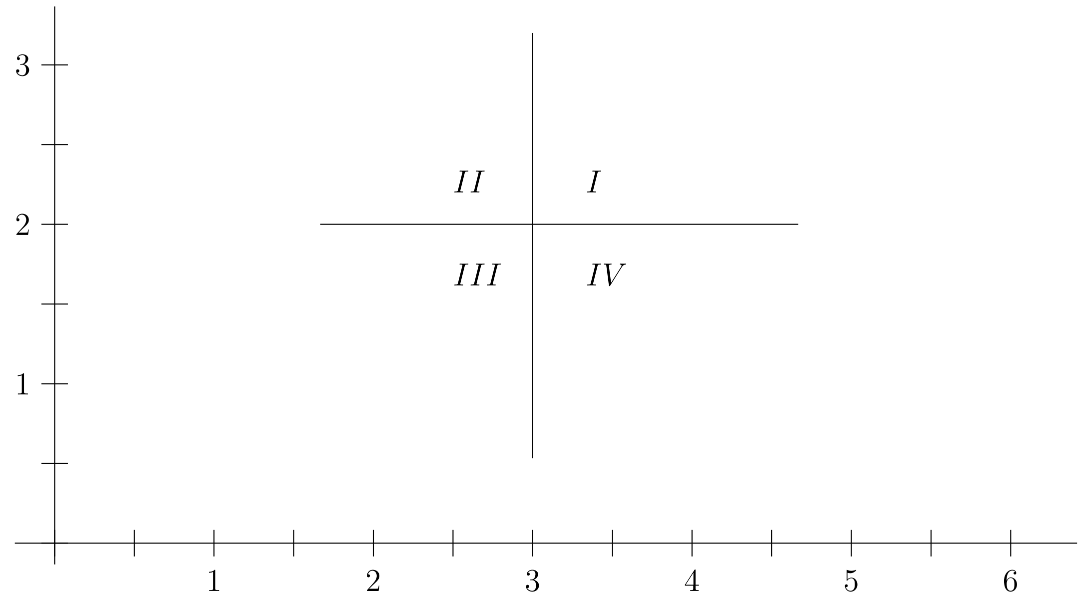
M146 Quiz 09 Thurs May 25, 2006 Name: ....

(1) We want an approximate solution to $\begin{array}{l} x' = x(2-y) \\ y' = y(x-3) \end{array}$ with $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$.

(1a) Do one step, with $\Delta t = 0.5$, and find approximate value for $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x(0.5) \\ y(0.5) \end{bmatrix}$.

(1b) Do a second step, with $\Delta t = 0.5$, and find approximate value for $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x(1) \\ y(1) \end{bmatrix}$.

(1c) Let $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ be the solution to $\begin{array}{l} x' = x(2-y) \\ y' = y(x-3) \end{array}$ with $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$.
The lines $x = 3$, and $y = 2$ partition the plane into four regions I, II, III, IV as shown (ignore the $x-$axis and the $y-$axis). The trajectory starts in region I. What are the next four regions through which the trajectory passes?



(1d) Using the calculations from **(1a)** **(1b)**, plot the points $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$, $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ and $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$.

(2) A drug D is taken into the stomach (S); it then goes into the bloodstream (B), and is then removed from (B) by the kidneys (K), where it stays.

Drug D goes from S to B at the continuous (i.e.intrinsic) rate of 30 % per hour.

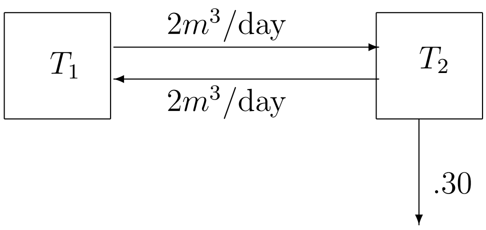
Drug D goes from B to K at the continuous (i.e.intrinsic) rate of 10 % per hour.

100 mg of drug D are taken at time $t = 0$. Let $x(t)$ be the amount of D in the stomach at time t, and let $y(t)$ be the amount of D in the blood at time t.

(2a) Write a system of differential equations for $x(t)$ and $y(t)$.

(2b) Solve this system for $(x(t), y(t))$ for all t.

(3) Two tanks T_1 and T_2 are connected as in the following diagram. Each tank contains 10 m^3 of liquid. The hoses connecting the two tanks carry 2 m^3 of liquid per day each way.



(3a) There is some bacteria in the two tanks. Tank T_2 is exposed to ultraviolet radiation that kills the bacteria at the continuous rate of 30% of the bacteria per day. Let $x_1(t)$ be the quantity of bacteria in T_1, and $x_2(t)$ be the quantity of bacteria in T_2. Write a system of differential equations for x_1 and x_2.

(3b) Initially T_1 contains 100 units of bacteria, and T_2 contains no bacteria. Calculate the amount of bacteria in each tank for all t.

M146 Final Monday, June 5, 2006

(1) Substance S is taken into the stomach, is absorbed by the blood, and removed from the blood by the kidney. The half-life of S in the stomach is 1 hour. The half-life of S in the blood is 5 hours. Initially there are 100 mg of S in the stomach, none in the blood.

(1a) Calculate the amount of S in the blood for all times t.

(1b) What is the maximum amount of S in the blood, and at what time does this maximum occur?

(2a) Find the steady state values for the differential equation $y' = 2\,y(y-5)(100-y)$. For each steady state value determine whether it is stable, or unstable..

(2b) Suppose $y = f(t)$ solves **(2a)** with $f(0) = 4$. What is $\lim_{t\to\infty} f(t)$?

(2c) Suppose $y = g(t)$ solves **(2a)** with $g(0) = 6$. What is $\lim_{t\to\infty} g(t)$?

(2d) In the absence of predators, a population of rabbits grows exponentially and would double in 7 months. A pack of foxes eat rabbits at a rate of 10 rabbits per month. Initially there are 120 rabbits. Calculate the number of rabbits for all times t.

(2e) Which happens? (A) All rabbits get eaten.

OR (B) The population of rabbits increases without limit.

If answer is (A), at what time does it occur?

If answer is (B), at what time does the population reach twice its original size?

(3) $A = \begin{bmatrix} 1 & 3 & 4 & -1 \\ 2 & 6 & 9 & 3 \\ 1 & 3 & 5 & 4 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$.

(3a) What is the rank of A? What is the nullity of A?

(3b) Find all solutions of $Ax = 0$.

(3c) Find all solutions of $Ax = b$.

(4) The table shows the length-weight relation for a species of Halibut. Use the method of least squares to find a power function $W = \alpha L^\beta$ which best fits the data.

L	0.4	1.0	2.5
W	1	10	100

(5) A population is divided into newborns, yearlings and adults. Each adult produces 2.2 newborns per year. Each year 55 % of the newborns survive to be yearlings, 70 % of yearlings survive to become adults and 40 % of the adults survive to the next year. After many generations the percentages of newborns, yearlings and adults in the population stabilizes. Find the stable percentages of newborns, yearlings and adults. (The largest eigenvalue for the Leslie matrix is 1.1)

(6) A certain ecosystem consists of A (= atmosphere), W (= water) and P = (plants), each of which contains sulfuric acid (SO_2). No SO_2 enters or leaves the ecosystem. From measurements, we find that

20 % of the SO_2 in the Air is transferred to the Water;
10 % of the SO_2 in the Air is transferred to the Plants;

30 % of the SO_2 in the Water is transferred to the Air;
10 % of the SO_2 in the Water is transferred to the Plants;

30 % of the SO_2 in the Plants is transferred to the Air;
20 % of the SO_2 in the Plants is transferred to the Water;

Find the stationary state distribution of 1200 lbs of SO_2.

(7a) $u = 3 + 4i, \quad v = 3 + i, \quad w = \dfrac{u}{v},$ Express w in rectangular from. Plot u, v and w in the cpmplex plane.

(7b) $a = 2 + i \quad b = -4 + 2i, \quad c = a \cdot b$ express a, b and c in polar form. Plot a, b and c in the cpmplex plane.

(7c) Find all complex numbers z which satisfy $z^3 = -8i$. Plot these in the complex plane.