(1) Find the steady state values for the following equations, and for each steady state value determine whether it is stable or unstable.

(1a) $\dfrac{dy}{dt} = 2(y-3)$

(1b) $\dfrac{dy}{dt} = 5(6-y)$

(1c) $\dfrac{dy}{dt} = 2\,y(y-1)$

(1d) $\dfrac{dy}{dt} = y(y^2-3)$

(1e) $\dfrac{dy}{dt} = .8\,y(y-.7)^2$

(2) Same equations as **(1)** . In each case, determine the behavior as $t \to +\infty$ of the solution with the initial condition listed. Do this without solving the equation for y.

(2a) $\dfrac{dy}{dt} = 2(y-3) \quad y(0) = 4$

(2b) $\dfrac{dy}{dt} = 5(6-y) \quad y(0) = 2$

(2c) $\dfrac{dy}{dt} = 2\,y(y-1)\,, \qquad y(0) = .5$

(2d) $\dfrac{dy}{dt} = 2\,y(y-1)\,, \qquad y(0) = 2$

(2e) $\dfrac{dy}{dt} = y(y^2-3)\,, \qquad y(0) = 1.5$

(2f) $\dfrac{dy}{dt} = .8\,y(y-.7)^2\,, \qquad y(0) = .5$

M146 Sample Quiz #01 for Thursday, March 30, 2006

(1) Find the steady state values for the following equation, and for each steady state value determine whether it is stable, or unstable.

$$\frac{dy}{dt} = 2\,y(y^2 - 4)(100 - y)$$

(1b) If $f(0) = 1$, what is $\lim_{t\to\infty} f(t)$?

(1c) If $f(0) = 10$, what is $\lim_{t\to\infty} f(t)$?

(1d) If $f(0) = 200$, what is $\lim_{t\to\infty} f(t)$?

(2a) Find the solution $\frac{dy}{dt} = -2y + 10\,, \qquad y(0) = 4\,.$

(2b) What is $\lim_{t\to\infty} y(t)$?

(2c) At what time does y reach 90 % of its steady state value?

(3a) $y = f(t)$ is the solution to $\frac{dy}{dt} = -.2y(1 - y)$, with $y(0) = 0.5$. What is $\lim_{t\to\infty} f(t)$?

(3b) Use separation of variables to solve the differential equation of **(3a)** .

(3c) At what time does y reach 95 % of its steady state value?

M146 Sample Quiz #02a for Thursday, April 6, 2006

(0) Atoms X radioactively decay to atoms Y, which in turn decay to atoms Z. The amount of X at time (in days) t is $100e^{-2t}$. The half-life of atoms Y is 20 days. Initially there are no atoms of Y or Z.

(0a) What is the rate at which X is decaying to Y?

(0b) What is the intrinsic decay rate for atoms Y?

(0c) Write a differential equation for y', i.e., the rate at which y is changing.

(0d) Solve this equation to calculate the amount of Y for all times t.

(1) Find the solution to $\dfrac{dy}{dt} = 3(y-1)t^2$, with $y(0) = \dfrac{1}{2}$.

(2) Solve the differential equation $y' = -2y + 100e^{-4t}$, with $y(0) = 0$.

(3a) A population of rabbits has an instantaneous annual growth rate (birth rate minus natural death rate) of .17 in the absence of predators. Foxes eat rabbits at a rate of 17 rabbits per fox per year. The initial rabbit population is 1750, and there are 20 foxes. Calculate the number of rabbits for all times t. (Assume the number of foxes stays constant.)

(3b) A population of rabbits has an instantaneous annual growth rate (birth rate minus natural death rate) of .17 in the absence of predators. Foxes eat rabbits at a rate of 17 rabbits per fox per year. The initial rabbit population is 1750. How many foxes should you introduce so that the population of rabbits remains constant. (Assume the number of foxes stays constant.)

(4a) The half-life of Sr-90 is 29 years. Suppose Sr-90 is introduced into the environment at the continuous rate of 2 gm per day (730 gm per year). Initially there is no Sr-90. Calculate the amount $y(t)$ of Sr-90 for all times t.

(4b) What is the limiting amount of Sr-90 in the environment?

(5) Radioactive substance X decays to Y with a half-life of 1 day. Y is also radioactive and decays to Z with a half-life of 3 days. Initially there is 100 grams of X and no grams of Y. Find the amount of Y for all times t.

M146 Sample Quiz #02b for Thursday, April 6, 2006

(1) Find the solution to $t^2 \dfrac{dy}{dt} = 2(y-1)^2$, with $y(1) = 3$.

(2) Solve the differential equation $y' = -3y + 100e^{-t}$, with $y(0) = 0$.

(3) A substance S is taken into the stomach, is absorbed into the blood, and then goes from the blood to the liver. S goes from the stomach to the blood at the (countinous) rate of 40 % per hour. S is removed from the blood at the continuous rate of 10 % per hour. Initially 100 mg of the S are present in the stomach, and 20mg in the blood. Let $y(t)$ be the amount of S in the blood at time t.

(3a) Write a differential equation for the rate of change of y.

(3b) Fnd the amount of S in the blood for all times t.

(3c) What is the maximum amount of S in the blood and at what time does it ocur?

(4a) A certain drug is administered intravenously at the rate of 2 mg per hour. The drug is excreted from the body at the (continuous) rate of 20 % per hour. Initially there is no drug in the body. Find the amount of the drug in the body for all times t.

(4b) How long does it take for the amount of the drug in the body to reach 90 % of its limiting value?

(5) A substance S is taken into the stomach, is absorbed into the blood, and then goes from the blood to the liver. S goes from the stomach to the blood at the (countinous) rate of 20 % per hour. S is removed from the blood at the continuous rate of 30 % per hour. In addition some more substance S is injected directly into the bloodstream at the constant rate of 5 mg per hour. Initially 100 mg of the S are present in the stomach, and none in the blood. Let $y(t)$ be the amount of S in the blood at time t. Write a differential equation for the rate of change of y. Fnd the amount of S in the blood for all times t.

M146 Sample Quiz #03 for Thurs, April 13, 2006

(1) Solve the differential equation $y' = -4y + 100e^{-3t} + 2$, with $y(0) = 0$.

(2) A substance S is taken into the stomach, is absorbed into the blood, and then goes from the blood to the liver. S goes from the stomach to the blood at the (countinous) rate of 20 % per hour. S is removed from the blood at the continuous rate of 40 % per hour. In addition some more substance S is injected directly into the bloodstream at the constant rate of 6 mg per hour. Initially 100 mg of the S are present in the stomach, and none in the blood. Let $y(t)$ be the amount of S in the blood at time t. Write a differential equation for the rate of change of y. Fnd the amount of S in the blood for all times t.

(3) Reduce $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & 5 & 12 \end{array}\right]$ to echelon form; then find all solutions of

$$\begin{array}{rcrcr} x_1 & + & x_2 & = & 3 \\ 2x_1 & + & 5x_2 & = & 12 \end{array}$$

(4) $A = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ and $b = \begin{bmatrix} 8 \\ 12 \end{bmatrix}$. Find all solutions of $Ax = b$.

(5) $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 9 \end{bmatrix}$ and $b = \begin{bmatrix} 6 \\ 13 \end{bmatrix}$. Find all solutions of $Ax = b$.

(6) $A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \\ 5 & 3 \end{bmatrix}$ $b = \begin{bmatrix} 6 \\ 8 \\ 7 \end{bmatrix}$. Find all solutions of $Ax = b$.

(1) Reduce $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & 5 & 7 \end{array}\right]$ to echelon form; then find all solutions of

$$\begin{array}{rcrcr} x_1 & + & x_2 & = & 3 \\ x_1 & + & 5x_2 & = & 7 \end{array}$$

(2) Reduce $\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 1 & -2 & -2 \end{array}\right]$ to echelon form; then find all solutions of

$$\begin{array}{rcrcr} x_1 & + & -x_2 & = & 1 \\ x_1 & + & -2x_2 & = & -2 \end{array}$$

(3) Reduce $\left[\begin{array}{ccc|c} -1 & -2 & 3 & -9 \\ 2 & 1 & -1 & 5 \\ 4 & -3 & 5 & -9 \end{array}\right]$ to echelon form; then find all solutions of

$$\begin{array}{rcrcrcr} -x_1 & + & -2x_2 & + & 3x_3 & = & -9 \\ 2x_1 & + & x_2 & + & -x_3 & = & 5 \\ 4x_1 & + & -3x_2 & + & 5x_3 & = & -9 \end{array}$$

(4a) Find all solutions of $\begin{array}{rcrcr} 2x_1 & + & 4x_2 & = & 4 \\ 3x_1 & + & 6x_2 & = & 1 \end{array}$

(4b) Find all solutions of $\begin{array}{rcrcr} 2x_1 & + & 4x_2 & = & 0 \\ 3x_1 & + & 6x_2 & = & 0 \end{array}$

(5) Find all solutions of $\begin{array}{rcrcrcr} 3x_1 & + & 4x_2 & + & 2x_3 & = & 2 \\ 6x_1 & + & 9x_2 & + & 5x_3 & = & 6 \end{array}$

(6) A population is divided into calves, yearlings, adults. The Leslie matrix for this population distribution is: $L = \begin{bmatrix} 0 & 0 & 2.5 \\ 0.6 & 0 & 0 \\ 0 & 0.4 & 0.4 \end{bmatrix}$.

(6a) Describe the meaning of each of the numbers in this matrix.

(6b) One eigenvalue (the largest) for L is $\lambda = 1.0$. Find a vector $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, with $x_1 + x_2 + x_3 = 1000$ so that $Lx = 1.0x$

(6c) A vector of numbers $p = (p_1, p_2, p_3)$ with $Lp = p$ and $p_1 + p_2 + p_3 = 1$ is called the **stable distribution**. What are the percentages of calves, yearlings and adults in the stable distibution?

M146 Sample #04 for Quiz April 20, 2006 Name: ....

(1) find all solutions of
$$\begin{array}{rcrcr} x_1 & + & 2x_2 & = & 1 \\ 2x_1 & + & 7x_2 & = & 8 \end{array}$$

(2) $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Calculate $A \cdot b$.

(3) Find all solutions of
$$\begin{array}{rcrcr} 1x_1 & + & 2x_2 & = & 4 \\ 3x_1 & + & 6x_2 & = & 1 \end{array}$$

(4) Find all solutions of
$$\begin{array}{rcrcr} 1x_1 & + & 2x_2 & = & 0 \\ 3x_1 & + & 6x_2 & = & 0 \end{array}$$

(5a) Let $A = \begin{bmatrix} 1 & -1 & 4 \\ -2 & 1 & 3 \\ 3 & 2 & -5 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$. Calculate $A \cdot b$.

(5b) Let $A = \begin{bmatrix} 1 & -1 & 4 \\ -2 & 1 & 3 \\ 3 & 2 & -5 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$. Find all solutions to $Ax = b$.

(6) A population is divided into calves, yearlings, adults. The Leslie matrix for this population distribution is: $L = \begin{bmatrix} 0 & 0 & 2.5 \\ 0.5 & 0 & 0 \\ 0 & 0.4 & 0.5 \end{bmatrix}$.

(6a) Describe the meaning of each of the numbers in this matrix.

(6b) The inutial population distribution is $\begin{bmatrix} 100 \\ 100 \\ 200 \end{bmatrix}$. What happens after 1 year? 2 years?

(6c) One eigenvalue (the largest) for L is $\lambda = 1.0$. Find a vector $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, with $x_1 + x_2 + x_3 = 760$ so that $Lx = 1.0x$

M146 Sample Quiz #05 for Thursday, April 27, 2006

(1) $A = \begin{bmatrix} 1 & 1 & -6 & 1 & -1 \\ 1 & -2 & 0 & -1 & -5 \\ 0 & 1 & -2 & 1 & 2 \\ 2 & 0 & -8 & 1 & 1 \end{bmatrix}$ $\quad \vec{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

What is the rank of A? What is the nullity of A? Give all solutions to $Ax = \vec{0}$.

(3) Let $v = \vec{AB}$, the vector from A to B. Plot v on the graph. Calculate a unit vector in the same direction as v. Give the equation for the line through A and B in parametric form.

(3) Let $A = \begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ -9 \end{bmatrix}$, $\vec{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Find all solutions of $Ax = \vec{0}$. Find all solutions of $Ax = b$. The solutions to $Ax = b$ can be described as the points on a line L. Give the equation for L is parametric form. Plot the line.

(4) $A = \begin{bmatrix} 1 & 6 \\ -2 & 9 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(5) Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 7 & -3 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Find all solutions of $Ax = 0$. Find all solutions of $Ax = b$. The solutions to $Ax = b$ can be described as the points on a line L. Give the equation for L is parametric form.

M146 Sample Quiz #06 for Thursday, May 4, 2006

(1) Let $C = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$. Find the point on the line $x = Ct$ closest to the point $b = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$.

(2) $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 5 & 1 \end{bmatrix}$; $b = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$. Find the best (least squares) solution to $Ax = b$

(3) The following data were obtained for the length y in centimeters of a human fetus versus the age t in weeks. Find a linear function $y = b + mt$ which best fits the data.

Age t	12	20	28	40
Length y	10	25	38	53

(4) Radioactive sample Y is decaying exponentially. The data shows the amount (y in grams) of Y at times (t in days). Find an exponential function $y = Ae^{rt}$ which best fits the data.

t	0	1	2	3	4	5
y	100	82	67	55	45	37

(5) The table shows the length-weight relation for Pacific halibut. Find an power function $W = \alpha L^{\beta}$ which best fits the data.

L	0.5	1.0	1.5	2.0	2.5
W	1.3	10.4	35	82	163

(6) The velocity of an enzymatic reaction with Michaelis-Menton kinetics is given by

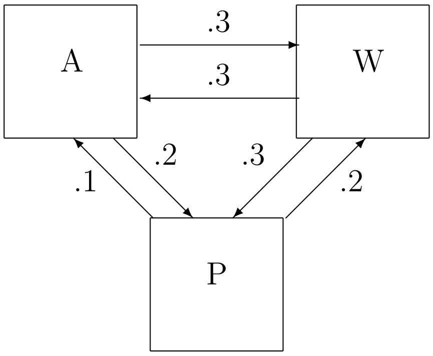
$$v(s) = \frac{\alpha s}{1 + \beta s}$$

Find the Michaelis-Menton equation which best fits the data:

s	1	2.5	5	10	20
v	4.1	6.1	9.3	12.9	17.1

M146 Sample Quiz #07 for Thursday, May 11, 2006

(1) The following diagram shows the transfer per year of S between three compartments A, W, and P.



Give the transfer matrix for this compartment model.

(1b) Initially there are 1200 tons of substance S in compartment A. Find the steady state distribution of S.

(2) $T = \begin{bmatrix} .8 & 0 & 0 \\ .1 & .6 & .2 \\ .1 & .4 & .8 \end{bmatrix}$ is the transfer matrix for a three compartment model.

Initially there are 1200 tons of substance S in compartment #1. Find the stable distribution.

(3) T is the 5 by 5 matrix for a 5 compartment model. What is the intermpretation of the number $T_{4,3}$.

(4) $T = \begin{bmatrix} .8 & 0 & .1 & .2 \\ .1 & 1 & .2 & .2 \\ .1 & 0 & .6 & .1 \\ 0 & 0 & .1 & .5 \end{bmatrix}$ is the transfer matrix for a four compartment model.

Initially there are 1000 tons of substance S in compartment #1. Find the stable distribution.

(5) $A = \begin{bmatrix} 3 & 1 \\ -3 & 7 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(6) Suppose T is a square matrix for which every column sum is 1. Explain why there is a non-zero vector X with $TX = X$.

M146 Sample Quiz #08 for Thursday, May 18, 2006

(1) Let $A = \begin{bmatrix} -3 & 0 \\ 4 & 2 \end{bmatrix}$

(1b) Find the eigenvalues and eigenvectors for the matrix A.

(1c) Find the general solution to the system $x' = AX$.

(1d) Find the solution to the system for which $x_1(0) = -5, \quad x_2(0) = -5$.

(2a) $A = \begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(2b) Find the general solution to the system $x' = AX$.

(2c) Find the solution to the system for which $x_1(0) = 6, \quad x_2(0) = 14$.

(3a) $A = \begin{bmatrix} -.3 & .2 \\ .1 & -.2 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(3b) Find the general solution to the system $x' = AX$.

(3c) Find the solution to the system for which $x_1(0) = 10, \quad x_2(0) = 0$.

(4a) $A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{bmatrix}$ The three eigenvalues are $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 3$.

Find three solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(4b) Find the general solution to the system $x' = AX$.

(4c) Find the solution to the system for which $x_1(0) = -5, \quad x_2(0) = -1.\ x_3(0) = 2$.

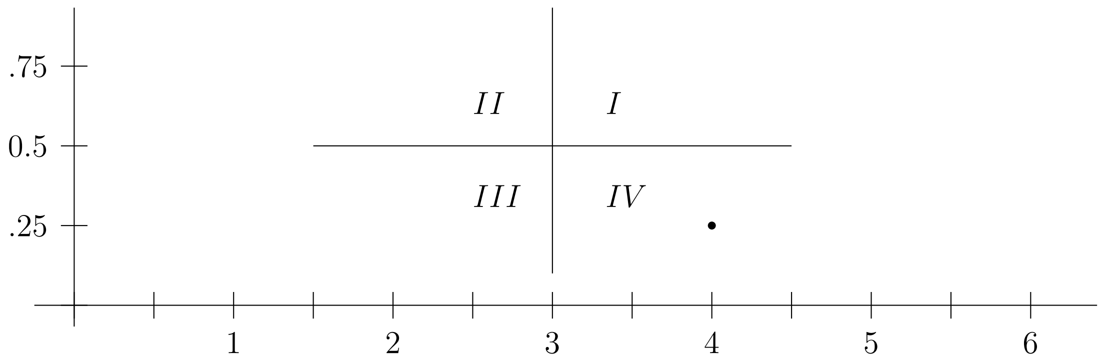
M146 Sample Quiz #09 for Thursday, May 25, 2006

(1) We want an approximate solution to $\begin{matrix} x' = x(-0.5+y) \\ y' = y(1.5-0.5x) \end{matrix}$ with $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 4 \\ 0.25 \end{bmatrix}$.

(1a) Take $\Delta t = 0.2$ and find approx. values for $(y(0.2), y(0.2))$.

(1b) Do two steps, each with $\Delta t = 0.2$, and find approx. values for $(y(0.4), y(0.4))$.

(1c) The lines $x = 3$, and $y = 0.5$ partition the plane into four regions I, II, III, IV as shown (ignore the $x-$axis and the $y-$axis). The trajectory of **(1b)** starts in region IV. What are the next four regions through which the trajectory passes?



(2) $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(2b) Find the general solution to the system $x' = AX$.

(2c) Find the solution to the system for which $x_1(0) = 0, \quad x_2(0) = 100$.

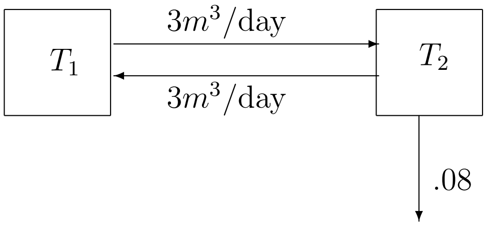
(3) A drug D is taken into the stomach; it then goes into the bloodstream, and is then removed from the bloodstream by the kidneys, and sent to the baldder, where it stays. The half-life of drug D in the stomach is 2 hours; the half-life of drug D in the bloodstream is 6 hours. 100 mg of drug D are taken at time $t = 0$. Let $x(t)$ be the amount of D in the stomach at time t, and let $y(t)$ be the amount of D in the blood at time t. let $z(t)$ be the amount of D in the bladder at time t. Draw a compartment diagram for x, y and z.

(3a) Write a system of differential equations for $x(t)$ and $y(t)$.

(3b) Solve this system for $(x(t), y(t))$ for all t.

(3c) What is the maximum amount of drug D that is ever present in the blood, and at what time does it occur?

(4) Two tanks T_1 and T_2 are connected as in the following diagram. Each tank contains 100 m^3 of liquid. The hoses connecting the two tanks carry 3 m^3 of liquid per day each way.



There is some bacteria in the two tanks. Tank T_2 is exposed to ultraviolet radiation that kills the bacteria at the continuous rate of 8% of the bacteria per day. Let $x_1(t)$ be the quantity of bacteria in T_1, and $x_2(t)$ be the quantity of bacteria in T_2. Write a system of differential equations for the quantities x_1 and x_2.

At $t = 0$, T_1 contains 100 units of bacteria, and T_2 contains no bacteria. Solve these equations for the amounts of bacteria in each tank for all t. What is the amount of bacteria remaining in T_1 after 10 days?

M146 Assignment X Due Friday, June 2, 2006

(1a) Verify $\frac{1+2i}{3-4i} + \frac{2-i}{5i} = -\frac{2}{5}$

(1b) Verify $\frac{5}{(1-i)(2-i)(3-i)} = \frac{1}{2}i$

(1c) Verify $(1-i)^4 = -4$

(2) Show that $z + \overline{z} = 2 \cdot \text{Re}z$.

(3) Let $z_1 = 4 + 3i$ and $z_2 = 5 + 12i$. Calculate $|z_1 z_2|$ and $|z_1||z_2|$.

(4) Find all complex numbers $z = x + iy$ such that $z^2 = 1 + i$.

(5) Let $z_1 = 3i$ and $z_2 = 2 - 2i$

(5a) Plot the points z_1, z_2 $z_1 + z_2$, $z_1 - z_2$ and $\overline{z_2}$.

(5b) Compute $|z_1 + z_2|$ and $|z_1 - z_2|$.

(5c) Express z_1 and z_2 in polar form.

(6) Show that $e^{i\theta_1}e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$.

(Use Euler's Formula for $e^{i\theta_1}$, $e^{i\theta_2}$ and the addition formulas for cos and sin.)

(7) Use (6) to show that $(e^{i\theta})^{-1} = e^{-i\theta}$, that is, Show that $e^{-i\theta}e^{i\theta} = 1$.

(8) Let $z_1 = 6e^{i\pi/3}$ and $z_2 = 2e^{-i\pi/6}$. Plot z_1, z_2, $z_1 z_2$ and $\frac{z_1}{z_2}$.

(9) Find all complex numbers z which satisfy $z^3 = -1$.

(10) Find all complex numbers $z = re^{i\theta}$ which satisfy $z^2 = \sqrt{2}e^{i\pi/4}$.