M146 Quiz 09 Thurs May 25, 2006 Name: ....

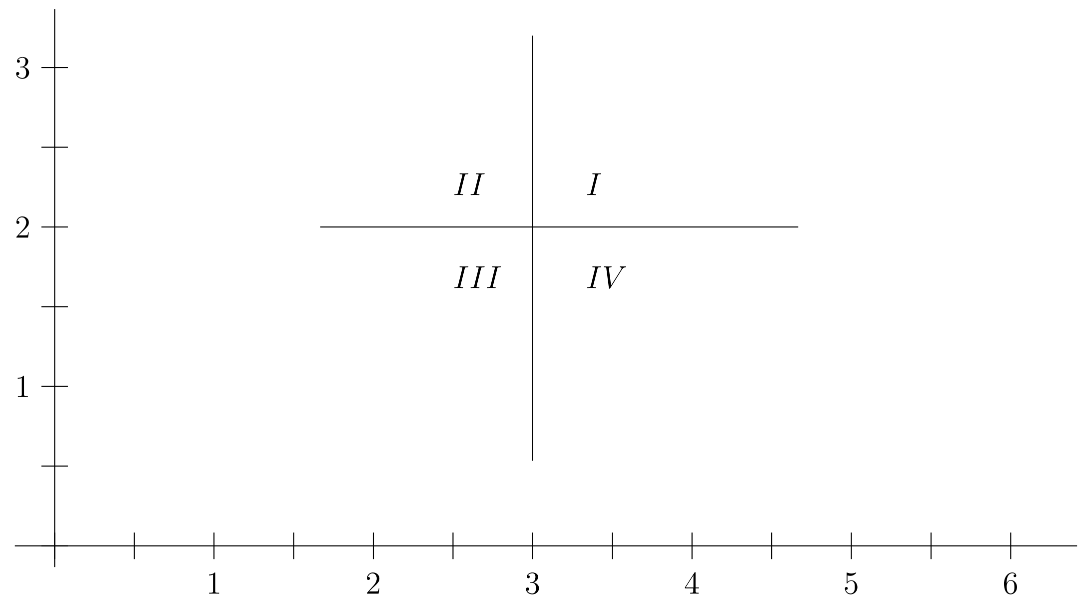
(1) We want an approximate solution to $\begin{array}{l} x' = x(2-y) \\ y' = y(x-3) \end{array}$ with $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$.

(1a) Do one step, with $\Delta t = 0.5$, and find approximate value for $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x(0.5) \\ y(0.5) \end{bmatrix}$.

(1b) Do a second step, with $\Delta t = 0.5$, and find approximate value for $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x(1) \\ y(1) \end{bmatrix}$.

(1c) Let $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ be the solution to $\begin{matrix} x' = x(2-y) \\ y' = y(x-3) \end{matrix}$ with $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$.

The lines $x = 3$, and $y = 2$ partition the plane into four regions I, II, III, IV as shown (ignore the $x-$axis and the $y-$axis). The trajectory starts in region I. What are the next four regions through which the trajectory passes?



(1d) Using the calculations from **(1a)** **(1b)**, plot the points $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$, $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ and $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$.

(2) A drug D is taken into the stomach (S); it then goes into the bloodstream (B), and is then removed from (B) by the kidneys (K), where it stays.

Drug D goes from S to B at the continuous (i.e.intrinsic) rate of 30 % per hour.

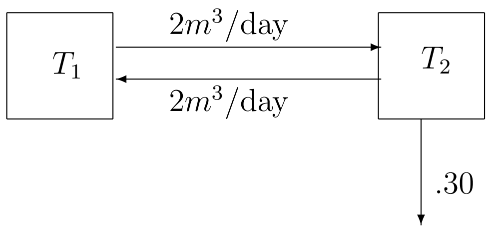
Drug D goes from B to K at the continuous (i.e.intrinsic) rate of 10 % per hour.

100 mg of drug D are taken at time $t = 0$. Let $x(t)$ be the amount of D in the stomach at time t, and let $y(t)$ be the amount of D in the blood at time t.

(2a) Write a system of differential equations for $x(t)$ and $y(t)$.

(2b) Solve this system for $(x(t), y(t))$ for all t.

(3) Two tanks T_1 and T_2 are connected as in the following diagram. Each tank contains 10 m^3 of liquid. The hoses connecting the two tanks carry 2 m^3 of liquid per day each way.



(3a) There is some bacteria in the two tanks. Tank T_2 is exposed to ultraviolet radiation that kills the bacteria at the continuous rate of 30% of the bacteria per day. Let $x_1(t)$ be the quantity of bacteria in T_1, and $x_2(t)$ be the quantity of bacteria in T_2. Write a system of differential equations for x_1 and x_2.

(3b) Initially T_1 contains 100 units of bacteria, and T_2 contains no bacteria. Calculate the amount of bacteria in each tank for all t.