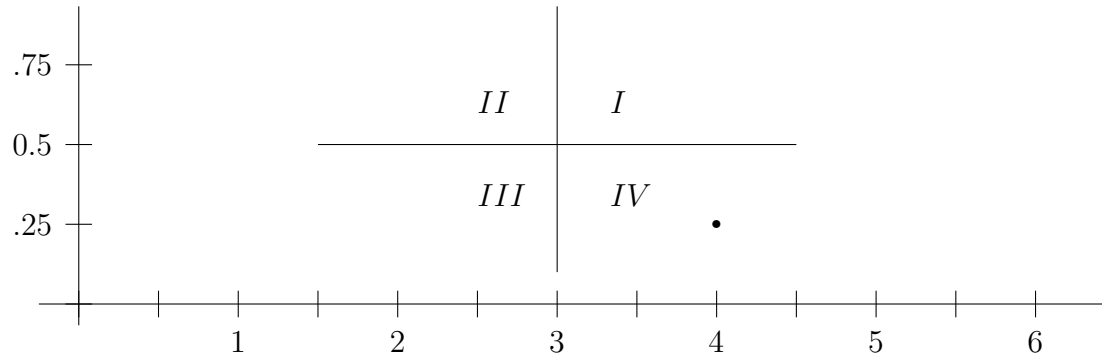
M146 Sample Quiz #09 for Thursday, May 25, 2006

(1) We want an approximate solution to $\begin{array}{c} x' = x(-0.5+y) \\ y' = y(1.5-0.5x) \end{array}$ with $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 4 \\ 0.25 \end{bmatrix}$.

(1a) Take $\Delta t = 0.2$ and find approx. values for $(y(0.2), y(0.2))$.

(1b) Do two steps, each with $\Delta t = 0.2$, and find approx. values for $(y(0.4), y(0.4))$.

(1c) The lines $x = 3$, and $y = 0.5$ partition the plane into four regions I, II, III, IV as shown (ignore the $x-$axis and the $y-$axis). The trajectory of **(1b)** starts in region IV. What are the next four regions through which the trajectory passes?



(2) $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

(2b) Find the general solution to the system $x' = AX$.

(2c) Find the solution to the system for which $x_1(0) = 0, \quad x_2(0) = 100$.

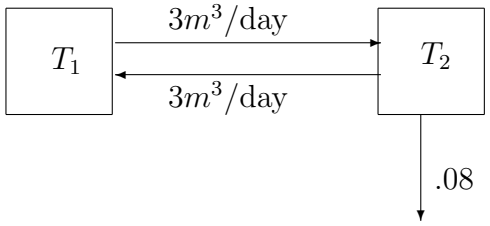
(3) A drug D is taken into the stomach; it then goes into the bloodstream, and is then removed from the bloodstream by the kidneys, and sent to the baldder, where it stays. The half-life of drug D in the stomach is 2 hours; the half-life of drug D in the bloodstream is 6 hours. 100 mg of drug D are taken at time $t = 0$. Let $x(t)$ be the amount of D in the stomach at time t, and let $y(t)$ be the amount of D in the blood at time t. let $z(t)$ be the amount of D in the bladder at time t. Draw a compartment diagram for x, y and z.

(3a) Write a system of differential equations for $x(t)$ and $y(t)$.

(3b) Solve this system for $(x(t), y(t))$ for all t.

(3c) What is the maximum amount of drug D that is ever present in the blood, and at what time does it occur?

(4) Two tanks T_1 and T_2 are connected as in the following diagram. Each tank contains 100 m^3 of liquid. The hoses connecting the two tanks carry 3 m^3 of liquid per day each way.



There is some bacteria in the two tanks. Tank T_2 is exposed to ultraviolet radiation that kills the bacteria at the continuous rate of 8% of the bacteria per day. Let $x_1(t)$ be the quantity of bacteria in T_1, and $x_2(t)$ be the quantity of bacteria in T_2. Write a system of differential equations for the quantities x_1 and x_2.

At $t = 0$, T_1 contains 100 units of bacteria, and T_2 contains no bacteria. Solve these equations for the amounts of bacteria in each tank for all t. What is the amount of bacteria remaining in T_1 after 10 days?