**M146 Sample Quiz #01a** for Friday, April 1, 2005

**(1)** Find the solution: $\dfrac{dy}{dt} = -.6\,y + 6\,, \qquad y(0) = 5\,.$ ß

**(2a)** A drug is administered at the rate of 6 mg per hour. The drug is excreted from the body at the (continuous) rate of 30 % per hour (i.e., the intrinsic decay rate is 0.30.) Initially there is no drug in the body. Calculate the amount of the drug in the body for all times $t$.

**(2b)** How long does it take for the amount of the drug in the body to reach 95 % of its limiting value?

**M146 Sample Quiz #01b** for Friday, April 1, 2005

**(1)** $y = f(t)$ is a function of $t$ which satisfies $y' = 0.1y(y-5)(40-y)$. Without finding the explicit solution to this differential equation, answer the following.

**(1a)** What are the critical values. For each critical value, state whether or not it is stable.

**(1b)** If $f(0) = 4$, what is $\lim_{t\to\infty} f(t)$?

**(1c)** If $f(0) = 10$, what is $\lim_{t\to\infty} f(t)$?

**(1d)** If $f(0) = 100$, what is $\lim_{t\to\infty} f(t)$?

**(2a)** At midnight 1000 gm of yeast begins to grow exponentially. At 5 AM there are 1649 gm. At noon, an inhibitor is added to the yeast which kills the yeast at the rate of 500 gm per hour. Calculate the amount of yeast at each time $t$ after noon.

**(2b)** Does all of the yeast eventually die, and if so, at what time?

**(3)** Find the solution: $\dfrac{dy}{dt} = 2ty^2, \qquad y(0) = 2\,.$

**M146 Sample Quiz #02** for Thursday, April 7, 2005

**(1)** Find the steady state values for the following equation, and for each steady state value determine whether it is stable, or unstable.

$$\frac{dy}{dt} = 2\,y(y^2 - 4)(100 - y)$$

**(1b)** If $f(0) = 1$, what is $\lim_{t\to\infty} f(t)$?

**(1c)** If $f(0) = 10$, what is $\lim_{t\to\infty} f(t)$?

**(1d)** If $f(0) = 200$, what is $\lim_{t\to\infty} f(t)$?

**(2)** Suppose the variables $x$ and $y$ are both dependent on time $t$, and $x^2 + y^2 = 100$. At the moment when $x = 6$, $\frac{dx}{dt} = -5$. What is the value of $\frac{dy}{dt}$?

**(3)** A cancer cell is growing exponentially. Initially the volume is 10 cubic cm, and 4 years later it is 15 cubic cm. How fast is the diameter increasing when the volume is 20 cubic cm?

**(4)** Find the solution to $\frac{dy}{dt} = -.2y(1 - y)$, with $y(0) = .5$.

**(4b)** What is $\lim_{t\to\infty} y(t)$?

**(5a)** A population of rabbits has an instantaneous annual growth rate (birth rate minus natural death rate) of .17 in the absence of predators. Foxes eat rabbits at a rate of 50 rabbits per fox per year. If the initial rabbit population is 1750, how many foxes should you introduce so that the population of rabbits remains constant. (Assume the number of foxes stays constant.)

**(5b)** Consider the same problem as **(2a)**. How many foxes should you introduce if you want the rabbit population to disappear in exactly 5 years. (Your answer to this problem, as for the previous problem, will be a non-integer number of foxes, but don't worry about it.)

**M146 Sample Quiz #03** Thurs, April 14, 2005 **SOLUTIONS**

**(1a)** Solve the differential equation $y' = -.5y + 10e^{-.1t}$, with $y(0) = 0$.

$$
\begin{aligned}
y' + .5y &= 10e^{-.1t} \\
e^{.5t}(y' + .5y) &= 10e^{.4t} \\
e^{.5t}y &= 25e^{.4t} + C, \qquad C = -25 \\
e^{.5t}y &= 25e^{.4t} - 25 \\
y &= 25e^{-.1t} - 25e^{-.5t}
\end{aligned}
$$

**(1b)** What is the maximum value of $y(t)$ and at what time does this max value occur?

$y' = -2.5e^{-.1t} + 12.5e^{-.5t}$

$y' = 0 \quad$ when $\quad 2.5e^{-.1t} = 12.5e^{-.5t}, \quad 2.5e^{.4t} = 12.5, \quad e^{.4t} = 5,$

$t = \dfrac{\ln 5}{.4} \approx 4.024 \quad y(4024) \approx 13.33$

**(2a)** A certain drug is administered intravenously at the rate of 2 mg per hour. The drug is excreted from the body at the (continuous) rate of 20 % per hour. Initially there is no drug in the body. Find the amount of the drug in the body for all times $t$.

$$
\begin{aligned}
y' &= +2 - .2y \\
\frac{dy}{dt} &= -.2(y - 10) \\
\int \frac{dy}{y - 10} &= \int -.2dt \\
\ln(y - 10) &= -.2t + C \\
y - 10 &= Ae^{-.2t}, \qquad A = -10 \\
y &= 10 - 10e^{-.2t}
\end{aligned}
$$

**(2b)** How long does it take for the amount of the drug in the body to reach 90 % of its limiting value?

$y = 9 \quad$ when $\quad 9 = 10 - 10e^{-.2t}, \quad 1 = 10e^{-.2t},$

$t \approx 11.513$

**(3)** Radioactive substance $X$ decays to $Y$ with a half-life of 2 years. $Y$ is also radioactive and decays to $Z$ with a half-life of 5 years. Initially 100 grams of $X$ are present. Find the amount of $Y$ for all times $t$. What is the maximum amount of $Y$ that is ever present, and when does it occur?

$$\begin{aligned} x' &= -.3466x \\ x &= 100e^{-.3466t} \\ x' &= -34.66e^{-.3466t} \end{aligned}$$

$$\begin{aligned} y' &= -.1386y + 34.66e^{-.3466t} \\ y' + .1386y &= 34.66e^{-.3466t} \\ e^{.1386t}(y' + .1386y) &= 34.66e^{-.2029t} \\ e^{.1386t}y &= -\frac{34.66}{.2029}e^{-.2029t} + C, \qquad C = 166.7 \\ y &= -166.7e^{-.3466t} + 166.7e^{-.1386t} \end{aligned}$$

$$\begin{aligned} y' &= 0 \quad \text{when} \quad .3466e^{-.3466t} = .1386e^{-.1386t} \\ t &= 4.40, \quad y(4.4) \approx 54 \end{aligned}$$

**M146 Sample Quiz #04** Thurs, April 21, 2005

**(1)** find all solutions of
$$\begin{array}{rcrcr} x_1 & + & 2x_2 & = & 1 \\ 2x_1 & + & 7x_2 & = & 8 \end{array}$$

**(2)** $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ $\quad b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Calculate $A \cdot b$.

**(3)** Find all solutions of
$$\begin{array}{rcrcr} 1x_1 & + & 2x_2 & = & 4 \\ 3x_1 & + & 6x_2 & = & 1 \end{array}$$

**(4)** Find all solutions of
$$\begin{array}{rcrcr} 1x_1 & + & 2x_2 & = & 0 \\ 3x_1 & + & 6x_2 & = & 0 \end{array}$$

**(5)** Find all solutions of
$$\begin{array}{rcrcrcr} x_1 & + & 3x_2 & + & 2x_3 & = & 2 \\ 2x_1 & + & 7x_2 & + & 7x_3 & = & 5 \end{array}$$

**(6)** Let $A = \begin{bmatrix} 1 & -1 & 4 \\ -2 & 1 & 3 \\ 3 & 2 & -5 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$. Calculate $A \cdot b$.

**(6)** Find all solutions of
$$\begin{array}{rcrcr} 2x_1 & + & 4x_2 & = & 6 \\ 3x_1 & + & 5x_2 & = & 8 \\ 5x_1 & + & 3x_2 & = & 8 \end{array}$$

**M146 Sample Quiz #05** Thurs, April 28, 2005

**(1)** Let $A = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 1 & 9 \\ 3 & 2 & 13 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$.

Find all solutions of $Ax = b$. Find all solutions of $Ax = 0$.

**(2)** Let $A = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 1 & 9 \\ 3 & 2 & 13 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$. Find all solutions of $Ax = b$.

**(3)** A population is divided into calves, yearlings, adults. The Leslie matrix for this population distribution is: $L = \begin{bmatrix} 0 & 0 & 2.5 \\ 0.5 & 0 & 0 \\ 0 & 0.4 & 0.5 \end{bmatrix}$.

**(3a)** Describe the meaning of each of the numbers in this matrix.

**(3b)** What happens after 1 year if the initial population is $\begin{bmatrix} 1000 \\ 600 \\ 300 \end{bmatrix}$.

**(3c)** What happens after 1 year if the initial population is $\begin{bmatrix} 1000 \\ 500 \\ 400 \end{bmatrix}$. After 2 years?

**(4a)** A population is divided into newborns and adults. Each adult produces 3 newborns per year; 10 % of the newborns survive to become adults; each year 70 % of the adults survive to the next year. Give the Leslie matrix $L$ for this population.

**(4b)** One eigenvalue is 1. Find a stable population $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ That is $Lx = x$, and $x_1 + x_2 = 1200$.

**M146 Sample Quiz #06** for Thursday, May 05, 2005

**(1)** Find all solutions of $\begin{array}{ccccc} y & + & x & = & 5 \\ z & - & y & = & -2 \\ x & + & z & = & 3 \end{array}$

**(2)** $A = \begin{bmatrix} 0 & 0 & 1 & 2 & 3 \\ 1 & 2 & 0 & 1 & 2 \\ 1 & 2 & 1 & 3 & 5 \\ 1 & 2 & -1 & -1 & -1 \end{bmatrix}$

What is the rank of $A$?

What is the nullity of $A$?

Give all solutions to $Ax = 0$.

**(3)** A population is divided into calves, yearlings, adults. The Leslie matrix for this population distribution is: $L = \begin{bmatrix} 0 & 0 & 2.4 \\ 0.6 & 0 & 0 \\ 0 & 0.5 & 0.7 \end{bmatrix}$.

**(3a)** Describe the meaning of each of the numbers in this matrix.

**(3b)** One eigenvalue (the largest) for $L$ is $\lambda = 1.2$. Find a vector $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, with $x_1 + x_2 + x_3 = 1200$ so that $Lx = 1.2x$

**(3c)** There is a vector of numbers $p = (p_1, p_2, p_3)$ with $p_1 + p_2 + p_3 = 1$ which is stable, called the **stable distribution**. What are the percentages of calves, yearlings and adults in the stable distibution?

**M146 Sample Quiz #07** for Thursday, May 12, 2005

**(1)** Plot the points $A = (2, 3)$ and $B = (5, -1)$. Find the angle between $OA$ and $OB$. ß Let $v = \vec{AB}$, the vector from $A$ to $B$. Plot $v$ on the graph. Calculate a unit vector in the same direction as $v$.

**(2)** $A = (4, 1)$ and $B = (2, 4)$. and $C = (5, -1)$. Plot these points on the graph. Find the angle between $\vec{CA}$ and $\vec{CB}$. Give the equation for the line through $A$ and $B$ in parametric form.

**(3a)** Let $A = \begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ -9 \end{bmatrix}$. Find all solutions of $Ax = 0$. Find all solutions of $Ax = b$. The solutions to $Ax = b$ can be described as the points on a line $L$. Give the equation for $L$ is parametric form. Plot the line.

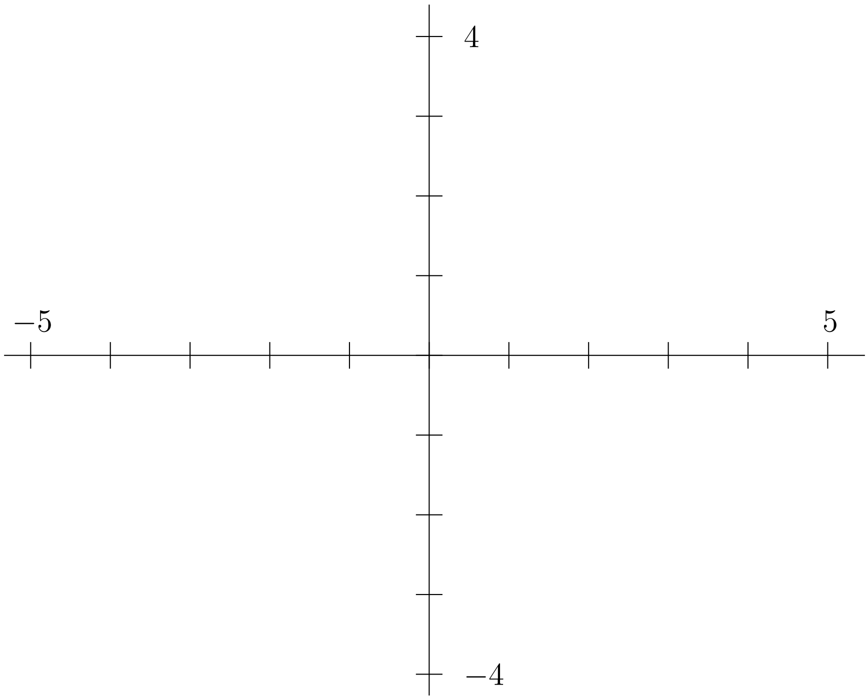
**(3a)** The vector $b = \begin{bmatrix} 3 \\ -9 \end{bmatrix}$ may be expressed as $b = x_1v_1 + x_2v_2$, where $v_1$ and $v_2$ are the columns of $A$. What are the values of $x_1$ and $x_2$?

**(4)** Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 7 & -3 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$. Find all solutions of $Ax = 0$. Find all solutions of $Ax = b$. The solutions to $Ax = b$ can be described as the points on a line $L$. Give the equation for $L$ is parametric form.

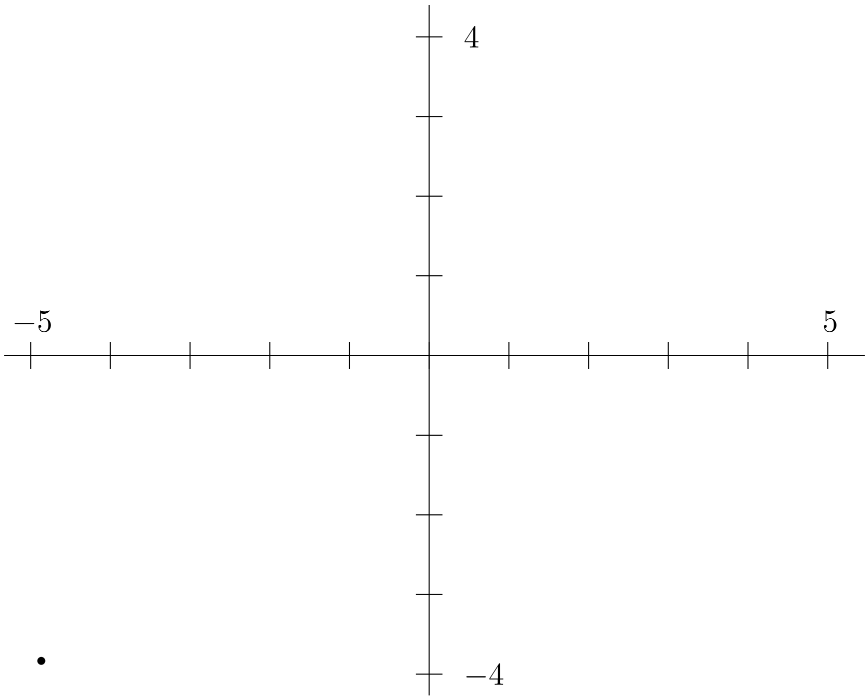
**(5)** $A = \begin{bmatrix} 2 & 3 \\ -1 & 6 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where $v$ is a vector and $\lambda$ is a scalar.

**M146 Sample Quiz #07a** for Thursday, May 12, 2005 **Solutions**

**(1)** Plot the points $A = (2, 3)$ and $B = (5, -1)$. Find the angle between $OA$ and $OB$. ß Let $v = \vec{AB}$, the vector from $A$ to $B$. Plot $v$ on the graph. Calculate a unit vector in the same direction as $v$.



**(2)** $A = (4, 1)$ and $B = (2, 4)$. and $C = (5, -1)$. Plot these points on the graph. Find the angle between $\vec{CA}$ and $\vec{CB}$. Give the equation for the line through $A$ and $B$ in parametric form.




**(3a)** Let $A = \begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ -9 \end{bmatrix}$. Find all solutions of $Ax = 0$. Find all solutions of $Ax = b$. The solutions to $Ax = b$ can be described as the points on a line $L$. Give the equation for $L$ is parametric form. Plot the line.

**(3a)** The vector $b = \begin{bmatrix} 3 \\ -9 \end{bmatrix}$ may be expressed as $b = x_1 v_1 + x_2 v_2$, where $v_1$ and $v_2$ are the columns of $A$. What are the values of $x_1$ and $x_2$?

**(4)** Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 7 & -3 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$. Find all solutions of $Ax = 0$. Find all solutions of $Ax = b$. The solutions to $Ax = b$ can be described as the points on a line $L$. Give the equation for $L$ is parametric form.

**(5)** $A = \begin{bmatrix} 2 & 3 \\ -1 & 6 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where $v$ is a vector and $\lambda$ is a scalar.

**M146 Sample Quiz #08** for Thursday, May 19, 2005

**M146 Sample Quiz #08** for Thursday, May 19, 2005