M146 Test #01 03/30/2006 **SOLUTIONS**

(1a) Find the steady state values for the differential equation $y' = 2\,y(y-3)(100-y)$. For each steady state value determine whether it is stable, or unstable..

$y = 0$ is a stable equillibrium point.

$y = 1$ is an unstable equillibrium point.

$y = 100$ is a stable equillibrium point.

(1b) Suppose $y = f(t)$ solves **(1a)** with $f(0) = 1$. Then $\lim_{t\to\infty} f(t) = 0$

(1c) Suppose $y = g(t)$ solves **(1a)** with $g(0) = 4$. Then $\lim_{t\to\infty} f(t) = 100$

(2a) Find the solution $\dfrac{dy}{dt} = -2y + 20\,, \qquad y(0) = 3\,.$

$$\begin{aligned}
\int \frac{dy}{y-10} &= -\int 2dt \quad \text{(separating variables)} \\
\ln(y-10) &= -2t + C \\
y - 10 &= Ae^{-2t} \quad 3 - 10 = A; \quad 7 = A \\
y &= 10 - 7e^{-2t}
\end{aligned}$$

(2b) At what time does y reach 90 % of its steady state value?

$9 = 10 - 7e^{-2t} \qquad 1 = 7e^{-2t}, \qquad t = .973$

(3a) Suppose $y = f(t)$ is the solution to $\frac{dy}{dt} = +0.1y(10 - y)$, with $y(0) = 1$. Without solving the equation, what is $\lim_{t\to\infty} f(t)$?

(There was a misprint in the problem; originally the + was a −.) The following gives the answers for the

The limit must be $y = 10$

(3b) Solve the differential equation $\frac{dy}{dt} = 0.1y(10 - y)$, with $y(0) = 1$.

$$y = \frac{10}{1 + 9e^{-.1t}}$$

(3c) At what time does y reach 50 % of its steady state value?

$$\begin{aligned} 5 &= \frac{10}{1 + 9e^{-.1t}} \\ 1 + 9e^{-.1t} &= \frac{10}{5} = 2 \\ 9e^{-.1t} &= 1 \\ e^{-.1t} &= \frac{1}{9} \\ -.1t &= -2.29 \\ t &= 22.9 \end{aligned}$$