M146 Test #02 04/06/2006 **SOLUTIONS**

(1) Find the solution to $\dfrac{dy}{dt} = 2(y-1)t \quad$ with $\quad y(0) = 3.$

$$\begin{aligned}
\int \frac{dy}{y-1} &= \int 2t dt \quad \text{(separating variables)} \\
\ln(y-1) &= t^2 + C \\
y - 1 &= Ae^{t^2} \qquad 3 - 1 = A; \qquad 2 = A \\
y &= 1 + 2e^{t^2}
\end{aligned}$$

(2a) Drug D is administered intravenously at the rate of 6.93 mg per hour. The amount of D in the body decreases exponentially; the half-life of D in the body is 3 hours. Initially there is no D in the body. Calculate the amount of D in the body for all times t.

$\dfrac{1}{2} = e^{r \cdot 3}$ so $r = -.231$

$$\begin{aligned}
\frac{dy}{dt} &= -.231y + 6.93 = -.231(y - 30) \\
\int \frac{dy}{y-30} &= -\int .231 dt \\
\ln(y-30) &= -.231t + C \\
y - 30 &= Ae^{-.231t} \\
0 - 30 &= A \\
y &= 30 - 30e^{-.231t}
\end{aligned}$$

(2b) At what time does the amount of D in the body to reach 90 % of its limiting value?

$$\begin{aligned}
27 &= 30 - 30e^{-.231t} \\
.1 &= e^{-.231t} \\
-2.30236 &= -.231t \\
t &= 9.97 \text{ hours}
\end{aligned}$$

(3) Substance S is taken into the stomach, is absorbed by the blood, then removed from the blood by the kidney. The intrinsic rate at which S goes from the stomach to the blood is 30 % per hour. The intrinsic rate at which S is removed from the blood is 10 % per hour. Initially 100 mg of the S are in the stomach, and none in the blood.

Let $x(t)$ be the amount of S in the stomach at time t.

Let $y(t)$ be the amount of S in the blood at time t..

(3a) Find the amount of S in the stomach at time t. Then calculate the rate at which S is leaving the stomach.

$$\begin{aligned} x &= 100e^{-.3t} \\ x' &= -30e^{-.3t} \end{aligned}$$

(3b) Write a differential equation for the rate of change of y. Then solve this to find the amount of S in the blood for all times t.

$$\begin{aligned} y' &= -.1y + 30e^{-.3t} \\ y' + .1y &= 30e^{-.3t} \\ e^{.1t}(y' + .1y) &= 30e^{-.2t} \\ e^{.1t}y &= \int 30e^{-.2t} = -\frac{30}{.2}e^{-.2t} + C \qquad\qquad C = 150 \\ y &= -150e^{-.3t} + 150e^{-.1t} \end{aligned}$$

(3c) At what time does the amount of S in the blood reach its maximum?

$$\begin{aligned} y' &= +45e^{-.3t} - 15e^{-.1t} \\ 45e^{-.3t} &= 15e^{-.1t} \\ 3 &= e^{.2t} \\ \ln 3 &= .2t \\ 5.493 &= t \end{aligned}$$