

M146 Test #03 04/13/2006 SOLUTIONS

(1) A substance S is taken into the stomach, is absorbed into the blood, then is removed from the blood by the liver. S goes from the stomach to the blood at the (continuous) rate of 30 % per hour. S is removed from the blood at the (continuous) rate of 20 % per hour. In addition, more S is injected directly into the bloodstream at the constant rate of 10 mg per hour. Initially 100 mg of S is present in the stomach, and none in the blood. Let $y(t)$ be the amount of S in the blood at time t . In the space below, answer **(1a)** - **(1c)**

(1a) Write a differential equation for the rate of change of y .

$$x'(t) = -30e^{-.3t}$$

$$y'(t) = +30e^{-.3t} + 10 - .2y$$

(1b) Solve **(1a)** to find the amount of S in the blood for all times t .

$$\begin{aligned} y'(t) &= +30e^{-.3t} + 10 - .2y \\ y'(t) + .2y &= +30e^{-.3t} + 10 \\ e^{.2t}(y'(t) + .2y) &= +30e^{-.1t} + 10e^{.2t} \\ (e^{.2t}y)' &= +30e^{-.1t} + 10e^{.2t} \\ (e^{.2t}y) &= -300e^{-.1t} + 50e^{.2t} + C \\ y &= -300e^{-.3t} + 50 + Ce^{-.2t} && C = 250 \\ y(t) &= -300e^{-.3t} + 50 + 250e^{-.2t} \end{aligned}$$

(1c) What is the limiting amount of S in the blood? Answer: $\lim_{t \rightarrow \infty} y(t) = 50$

(2) $A = \begin{bmatrix} 2 & 4 \\ 3 & 7 \end{bmatrix}$ and $b = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$. Find all solutions of $Ax = b$.

$$\left[\begin{array}{cc|c} 2 & 4 & 8 \\ 3 & 7 & 13 \end{array} \right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 3 & 7 & 13 \end{array} \right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 1 \end{array} \right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array} \right] \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

(3) $A = \begin{bmatrix} 2 & 4 \\ 3 & 7 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Find all solutions of $Ax = b$.

$$\left[\begin{array}{cc|c} 2 & 4 & 0 \\ 3 & 7 & 0 \end{array} \right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right] \quad \text{The only solution is } \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

(4) Find all solutions of
$$\begin{aligned} x_1 + x_2 + 3x_3 &= 2 \\ 2x_1 + 5x_2 + 12x_3 &= 7 \\ 3x_1 + 6x_2 + 15x_3 &= 9 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 2 & 5 & 12 & 7 \\ 3 & 6 & 15 & 9 \end{array} \right] \longrightarrow \dots \quad \text{reduces to} \quad \dots \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

The solutions are
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 - a \\ 1 - 2a \\ a \end{bmatrix}.$$