M146 Test #04 SOLUTIONS

(1) Let $A = \begin{bmatrix} 1 & -1 & 4 \\ -2 & 1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$. Find all solutions to $Ax = b$.

SOLUTION:

$$\left[\begin{array}{rrr|r} 1 & -1 & 4 & 1 \\ -2 & 1 & 3 & -3 \\ 3 & -2 & 1 & 4 \end{array}\right] \quad \text{reduces to} \quad \left[\begin{array}{rrr|r} 1 & 0 & -7 & 2 \\ 0 & 1 & -11 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The solutions are $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2+7a \\ 1+11a \\ a \end{bmatrix}$

(2a) A population is divided into newborns and adults. Each adult produces 2 newborns per year; 30 % of the newborns survive to become adults; each year 70 % of the adults survive to the next year. Give the Leslie matrix L for this population.

(2b) One eigenvalue (the largest) for L is $\lambda = 1.2$. Find a population distribution of newborns and adults which is unchanged in each generation.

That is, find $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ with $x_1 + x_2 = 1$ and $Lx = 1.2x$.

SOLUTION:

$$L = \begin{bmatrix} 0 & 2 \\ 0.3 & 0.7 \end{bmatrix}; \quad L - 1.2I = \begin{bmatrix} -1.2 & 2 \\ 0.3 & -0.5 \end{bmatrix} \quad \text{reduces to} \quad \begin{bmatrix} 1 & -\frac{5}{3} \\ 0 & 0 \end{bmatrix}$$

The eigenvector with $x_1 + x_2 = 1$ is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{8} \\ \frac{3}{8} \end{bmatrix}$

(3a) A population is divided into calves, yearlings, adults. The Leslie matrix for this population is $L = \begin{bmatrix} 0 & 0 & 2 \\ 0.3 & 0 & 0 \\ 0 & 0.5 & 0.7 \end{bmatrix}$. Give the meaning of each non-zero number in this matrix.

SOLUTION:

Each adult produces 2 newborns per year;

Each year 30 % of the newborns survive to become yearlings;

Each year 50 % of the yearlings survive to become adults.

Each year 70 % of the adults survive to the next year.

(3b) One eigenvalue (the largest) for L is $\lambda = 1.0$. Find a stable population which totals 360. That is find $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ with $x_1 + x_2 + x_3 = 360 \quad \text{and} \quad Lx = x.$

SOLUTION:

$$L - I = \begin{bmatrix} -1 & 0 & 2 \\ 0.3 & -1 & 0 \\ 0 & 0.5 & -0.3 \end{bmatrix} \quad \text{reduces to} \quad \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -\frac{3}{5} \\ 0 & 0 & 0 \end{bmatrix}$$

The solutions are $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{3}{5} \\ 1 \end{bmatrix} \cdot a$

$$\left(2 + \frac{3}{5} + 1\right)a = 360, \quad a = 100, \quad x = \begin{bmatrix} 200 \\ 60 \\ 100 \end{bmatrix}$$