M146 Test #05 Thursday, April 27, 2006 **Solutions**

(1) $B = (2, 4)$. and $C = (6, -2)$. Give the equation for the line through B and C in parametric form.

$\vec{BC} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$. $\quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \end{bmatrix} \cdot a + \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

(2) $A = \begin{bmatrix} 1 & 1 & -6 & 1 & -1 \\ 1 & -2 & 0 & -1 & -5 \\ 0 & 1 & -2 & 1 & 2 \\ 2 & 0 & -8 & 1 & 1 \end{bmatrix}$ $\quad \vec{b} = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 3 \end{bmatrix}$.

What is the rank of A? What is the nullity of A? Give all solutions to $Ax = \vec{b}$.

A reduces to $\begin{bmatrix} 1 & 0 & -4 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$; the rank is 4; the nullity is 5-4 = 1.

reducing $\vec{b}$ at the sane time gives $\vec{c} = \begin{bmatrix} 2.6 \\ 3 \\ -2.4 \\ 0.2 \end{bmatrix}$. The solutions are $\begin{bmatrix} 2.6 \\ 3 \\ 0 \\ -2.4 \\ 0.2 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \cdot a$.

(3) Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 7 & -3 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Find all solutions of $Ax = 0$. Find all solutions of $Ax = b$. The solutions to $Ax = b$ can be described as the points on a line L. Give the equation for L is parametric form.

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 3 & 7 & -3 & 4 \end{array}\right] \text{ reduces to } \left[\begin{array}{ccc|c} 1 & 0 & 2.5 & -0.5 \\ 0 & 1 & -1.5 & 0.5 \end{array}\right]$$

The solutions are $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2.5 \\ 1.5 \\ 1 \end{bmatrix} \cdot a + \begin{bmatrix} -0.5 \\ 0.5 \\ 0 \end{bmatrix}$.

(4) $A = \begin{bmatrix} 1 & 6 \\ -2 & 9 \end{bmatrix}$ Find two solutions to $Av = \lambda v$ where v is a vector and λ is a scalar.

$$\det \begin{bmatrix} 1-\lambda & 6 \\ -2 & 9-\lambda \end{bmatrix} = (1-\lambda)(9-\lambda) + 12 = \lambda^2 - 10\lambda + 21 = (\lambda - 3)(\lambda - 7)$$

$$\lambda_1 = 3: \qquad A - 3I = \begin{bmatrix} -2 & 6 \\ -2 & 6 \end{bmatrix}, \qquad v_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\lambda_1 = 7: \qquad A - 7I = \begin{bmatrix} -6 & 6 \\ -2 & 2 \end{bmatrix}, \qquad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$