

M146 Test #06 Thursday, May 04, 2006 **SOLUTIONS**

(1) . Find the point on the line $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \cdot t$ closest to the point $b = \begin{bmatrix} 5 \\ 5 \\ 7 \end{bmatrix}$.

$$C^T \cdot C = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = 1 + 1 + 4 = 6$$

$$C^T \cdot b = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 5 \\ 7 \end{bmatrix} = 5 + 5 + 14 = 24$$

$$6x = 24; \quad x = 4 \quad \text{Pt} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \cdot 4 = \begin{bmatrix} 4 \\ 4 \\ 8 \end{bmatrix}$$

(2) Find the best least squares solution to:

$$\begin{array}{rcl} -x_1 & + & 2x_2 = 6 \\ & & -2x_2 = 3 \\ 2x_1 & - & 6x_2 = 3 \end{array}$$

$$A^T \cdot A = \begin{bmatrix} -1 & 0 & 2 \\ 2 & -2 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 0 & -2 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 5 & -14 \\ -14 & 26 \end{bmatrix}$$

$$A^T \cdot b = \begin{bmatrix} -1 & 0 & 2 \\ 2 & -2 & 6 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -12 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 5 & -14 & 0 \\ -14 & 26 & -12 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & -2.8 & 0 \\ 0 & 1 & -2.5 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & -7 \\ 0 & 1 & -2.5 \end{array} \right];$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -7 \\ -2.5 \end{bmatrix}$$

(3) The following data were obtained for the length y in centimeters of a salamander versus the age t in weeks. Find a linear function $y = b + mt$ which best fits the data.

Age	t	1	3	4
Length	y	3	7	9

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}; \quad b = \begin{bmatrix} 3 \\ 7 \\ 9 \end{bmatrix}.$$

$$A^T \cdot A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 8 & 26 \end{bmatrix} \quad A^T \cdot b = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 7 \\ 9 \end{bmatrix} = \begin{bmatrix} 19 \\ 60 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 3 & 8 & 19 \\ 8 & 26 & 60 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & -2.67 & 6.33 \\ 0 & 4.67 & 9.33 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array} \right]; \quad \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(4) A colony of yeast Y is decaying exponentially. The data shows the amount (y in grams) of Y at times (t in days). Find an exponential function $y = Ae^{rt}$ which best fits the data.

t	1	5	10
y	12	27	74

$w = \ln y$	2.485	3.296	4.304
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$$A = \begin{bmatrix} 1 & 1 \\ 1 & 5 \\ 1 & 10 \end{bmatrix}; \quad w = \begin{bmatrix} 2.485 \\ 3.296 \\ 4.304 \end{bmatrix}.$$

$$A^T \cdot A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 10 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 5 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 3 & 16 \\ 16 & 126 \end{bmatrix}; \quad A^T \cdot b = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 10 \end{bmatrix} \cdot \begin{bmatrix} 2.485 \\ 3.296 \\ 4.304 \end{bmatrix} = \begin{bmatrix} 10.1 \\ 62 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 3 & 16 & 10.1 \\ 16 & 126 & 62 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 5.33 & 3.367 \\ 0 & 40.72 & 8.133 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2.3 \\ 0 & 1 & -.2 \end{array} \right]; \quad \begin{bmatrix} b \\ m \end{bmatrix} \approx \begin{bmatrix} 2.3 \\ .2 \end{bmatrix}$$

$$A = e^{2.3} \approx 10; \quad \text{the exponential is } y = 10e^{.2t}$$