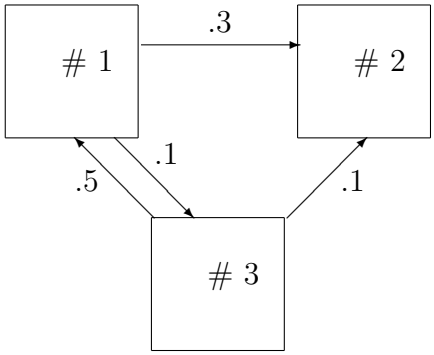
M146 Test #07 Thursday, May 11, 2006 **Solutions**

(1) $T = \begin{bmatrix} .6 & 0 & .5 \\ .3 & 1 & .1 \\ .1 & 0 & .4 \end{bmatrix}$ is the transfer matrix for a three compartment model. ($T_{1,3}$ corrected)

(1a) Make a 3 compartment diagram with arrows showing the transfer.



(1b) Initially there are 500 tons of substance S in compartment #1, 300 tons in # 2, and 200 tons in # 3. What is the stable distribution? Explain your answer. In particular how you know the stable distribution with (almost) no calculations.

Substance S enters # 2 but never leaves, so 500+300+200 =1000 is the stable distribution, all in # 2.

(2) $T = \begin{bmatrix} .6 & .2 & .2 \\ 0 & .7 & 0 \\ .4 & .1 & .8 \end{bmatrix}$ is the transfer matrix for a three compartment model.

Initially there are 1200 tons of substance S in compartment #1. Find the stable distribution.

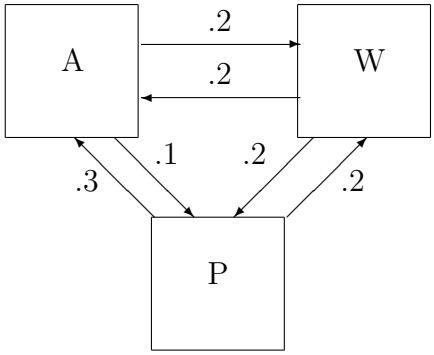
$$T-I = \begin{bmatrix} -.4 & .2 & .2 \\ 0 & -.3 & 0 \\ .4 & .1 & -.2 \end{bmatrix} \quad \text{reduces to} \quad \begin{bmatrix} 1 & 0 & -.5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{The eigenvector is } v = \begin{bmatrix} 0.5 \\ 0 \\ 1 \end{bmatrix}.$$

The solution is $X = \begin{bmatrix} 0.5a \\ 0 \\ a \end{bmatrix}$. with $1.5a = 1200$ $a = 800$; $X = \begin{bmatrix} 400 \\ 0 \\ 800 \end{bmatrix}$.

(3) T is the 5 by 5 matrix for a 5 compartment model. What is the interpretation of the number $T_{2,3}$.

$T_{2,3}$ is the percentage (as a decimal) of substance that goes from # 3 to # 2.

(4) The diagram shows the transfer per year of S between 3 compartments A, W, and P.



(4a) The transfer matrix for this compartment model. is $T = \begin{bmatrix} .7 & .2 & .3 \\ .2 & .6 & .2 \\ .1 & .2 & .5 \end{bmatrix}$

(4b) Initially there are 300 tons of substance S in compartment A, 100 in W, and 500 in P. Find the steady state distribution of S.

$$T - I = \begin{bmatrix} -.3 & .2 & .3 \\ .2 & -.4 & .2 \\ .1 & .2 & -.5 \end{bmatrix} \quad \text{reduces to} \quad \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1,5 \\ 0 & 0 & 0 \end{bmatrix}$$

The solution is $X = \begin{bmatrix} 2a \\ 1.5 \\ a \end{bmatrix}$, with $4.5a = 300 + 100 + 500 \quad a = 200; \quad X = \begin{bmatrix} 400 \\ 300 \\ 200 \end{bmatrix}$.