M308B New Sample Dec 6, 2007

(1) (10 points) A is a 8 by 11 matrix; A reduces to a matrix in RREF with 3 zero rows. What are the dimensions of the nullspace, rowspace and columnspace of A?

(2) $A = \begin{bmatrix} 1 & 1 & 0 & -2 & 3 \\ 1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 2 & 0 & -1 \\ 2 & 2 & 1 & -3 & 4 \end{bmatrix}$ **(2a)** Find a basis for the rowspace of A.

(2b) Find a basis for the nullspace of A.

(2c) Find a subset of the columns of A that are a basis for the column space of A.

(3) Calculate the determinant of $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$.

(4) Let T be the linear transformation of the plane to itself for which $T(1,2) = (1,4)$ and $T(3,7) = (5,2)$. Find $T(4,7)$.

(5a) Let $v = [2,1,3]$. Let T be the linear transformation of R^3 to itself which is the projection of any vector on the line through v. Find the matrix of T with respect to the standard basis.

(5b) What is the rank of T?

(6) $A = \begin{bmatrix} 10 & 3 \\ 3 & 2 \end{bmatrix}$. Find an invertible matrix S for which $S^T \cdot S = I$, and a diagonal matrix D with $S^T AS = D$.

(7a) $A = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & -4 \\ 2 & 1 & -1 \end{bmatrix}$. The characteristic polynomial for A is $f(t) = -(t-1)(t-2)(t-3)$. What are the eigenvalues and corresponding eigenvectors of A.

(7b) Find an invertible matrix S and a diagonal matrix D so that $S^{-1}AS = D$.

(8) $w = [6,3,3]$, and V is the plane spanned by $v_1 = [1,1,0]$ and $v_2 = [1,2,2]$. Find a vector x in V and a vector $y \perp V$ so that $w = x + y$.

(9) Find the line of best least squares fit to the data $(-2,-1)$ $(-1,1)$, $(1,5)$, $(2,7)$.

(10a) Let π be the plane perpendicular to the vector $[2,-1,2]$. Find a basis for π.

(10b) Find the projection of $b = (2,2,7)$ on π.