M308 Sample Problems #1

(1) $A = \begin{bmatrix} 1 & 2 & 1 & -1 & 1 \\ 2 & 4 & 1 & -4 & -3 \\ -1 & -2 & 1 & 5 & 5 \end{bmatrix}$ $\quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$ $\quad d = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$

(1a) Find all solutions of $Ax = 0$.

(1b) Find all solutions of $Ax = d$.

(1c) Is there a vector b for which $Ax = b$ has no solution x? If so, find one.

(2a) $A = \begin{bmatrix} 1 & 2 & 3 & 1 & 5 & 7 \\ 1 & 2 & 3 & 2 & 6 & 9 \\ -1 & -2 & -3 & 1 & -3 & -3 \end{bmatrix}$

(2b) For the matrix A of **(2a)**, find a basis for the rowspace.

(2c) Find a basis for the nullspace of A.

(2d) Find a basis for the columnspace of A.

(2e) Express $b = \begin{bmatrix} 5 \\ 8 \\ 1 \end{bmatrix}$ as a linear combination of the columns of A.

(3a) Given a set $\{v_1, \ldots, v_k\}$ of vectors in R^n, and another vector w, how can you decide if w is a linear combination of $v_1, \ldots v_k$?

(3b) $v_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, $w = \begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix}$. Is w a linear combination of v_1 and v_2?

(3c) $x = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$. Is x a linear combination of v_1 and v_2?

(4) A 3 by 3 matrix A reduces to the identity by the following row operations:

1) Add $2R_1$ to R_2
2) Add $-3R_1$ to R_3
3) Multiply R_3 by $1/2$
4) Add $-R_3$ to R_2
5) Add $-3R_2$ to R_1

(4a) What is A^{-1}?

(4b) Express A^{-1} as a product of elementary matrices.

(4c) Express A as a product of elementary matrices.

(4d) What is A?

(4e) Find the solution to $Ax = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$.

(5a) $B = \begin{bmatrix} 1 & 4 & 0 \\ 4 & 14 & 4 \\ 0 & 4 & 0 \end{bmatrix}$. Find the inverse of B.

(5b) Find the solution to $Bx = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$.

(6a) If $\{v_1, \ldots, v_k\}$ is a set of vectors in R^n, how can you tell if they are linearly dependent or linearly independent?

(6b) $v_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$. $v_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$. $v_3 = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$, $v_4 = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$. Are v_1, v_2 and v_3 linearly independent?

(6c) Are v_1, v_2 and v_4 linearly independent?

(7) $A = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -4 \end{bmatrix}$ Are the columns of A linearly independent?

(8a) What is the definition that a set of vectors $v_1, v_2, \ldots v_m$ span a vector space W?

(8b) Given a set $\{v_1, \ldots, v_k\}$ of vectors in R^n, and another vector w, how can you tell if $w \in$ span $(v_1, \ldots v_k)$?

(8c) $v_1 = (1, 3, 2)$, $v_2 = (2, 1, -1)$, and $w = (3, -1, -4)$. Is $w \in$ span (v_1, v_2)?

(9) A is a 9 by 13 matrix; A reduces to a matrix in RREF with 2 zero rows. What are the dimensions of the nullspace, rowspace and columnspace of A?

(10a) What is the definition that a set of vectors $v_1, v_2, \ldots v_m$ are a basis for W?

(10b) Suppose $v_1, \ldots, v_k$ span a vector space $\mathbf{V}$. How do you find a basis for $\mathbf{V}$?

(10c) $\mathbf{V}$ is spanned by $v_1 = (1, 1, 2)$, $v_2 = (2, 1, -1)$, $v_3 = (3, 1, -4)$. Find a basis for $\mathbf{V}$.

(11) $A = \begin{bmatrix} 1 & 2 & 3 & 1 & 6 & 5 \\ 1 & 2 & 3 & 2 & 9 & 7 \\ 2 & 4 & 6 & 1 & 9 & 8 \end{bmatrix}$ Find a basis for the rowspace of A

(11b) Find a basis for the nullspace of A.

(11c) Find a subset of the columns which form basis for the columnspace of A.

M308 Sample Problems #2

(1a) Given a set $\{v_1, \ldots, v_k\}$ of vectors in R^n, and another vector w, how can you decide if w is a linear combination of $v_1, \ldots v_k$?

(1b) $v_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, $w = \begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix}$. Is w a linear combination of v_1 and v_2?

(1c) $u = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$. Is u a linear combination of v_1 and v_2?

(3a) $B = \begin{bmatrix} 1 & 4 & 0 \\ 4 & 14 & 4 \\ 0 & 4 & 0 \end{bmatrix}$. Find the inverse of B.

(3b) Find the solution to $Bx = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$.

(4a) What is the definition that a set of vectors $v_1, v_2, \ldots v_k$ are linearly dependent?

(4b) What is the definition that a set of vectors $v_1, v_2, \ldots v_k$ are linearly independent?

(5a) If $\{v_1, \ldots, v_k\}$ is a set of vectors in R^n, how can you tell if they are linearly dependent or linearly independent?

(5b) $v_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$. $v_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$. $v_3 = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$, $v_4 = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$. Are v_1, v_2 and v_3 linearly independent?

(5c) Are v_1, v_2 and v_4 linearly independent?

(6a) What is the definition that a set of vectors $v_1, v_2, \ldots v_m$ span a vector space W?

(6b) Given a set $\{v_1, \ldots, v_k\}$ of vectors in R^n, and another vector w, how can you tell if $w \in$ span $(v_1, \ldots v_k)$?

(6c) $v_1 = (1, 3, 2)$, $v_2 = (2, 1, -1)$, and $w = (3, -1, -4)$. Is $w \in$ span (v_1, v_2)?

(7) Let A be an n by n matrix. Show that the following conditions are equivalent. (If any one of them is true so are all the others. If any one of them is false so are all the others.)

(i) A is row equivalent to the identity matrix.

(ii) For each vector b in R^n, there is exactly one solution to $Ax = b$.

(iii) The only solution to $Ax = 0$ is $x = 0$.

(iv) The columns of A are linearly independent.

(v) $\det A \neq 0$.

(vi) There is an matrix A^{-1}, with $A^{-1}A = I$. (Also $AA^{-1} = I$.)

(vii) The rows of A are linearly independent.

(vii) $\det(A) \neq 0$