

**M308 Sample Test #1** For Jan 31, 2007

(1)  $A = \begin{bmatrix} 1 & -2 & 1 & -2 & 2 \\ 1 & -2 & 2 & -5 & 6 \\ 2 & -4 & 1 & -1 & 1 \end{bmatrix}$ . Find all solutions of  $Ax = \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix}$ .

(2a) Given a set  $\{v_1, \dots, v_k\}$  of vectors in  $R^n$ , and another vector  $w$ , how can you decide if  $w$  is a linear combination of  $v_1, \dots, v_k$ ?

(2b)  $v_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ ,  $v_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ ,  $w = \begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix}$ . Is  $w$  a linear combination of  $v_1$  and  $v_2$ ?

(2c)  $u = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$ . Is  $u$  a linear combination of  $v_1$  and  $v_2$ ?

(3a) If  $\{v_1, \dots, v_k\}$  is a set of vectors in  $R^n$ , how can you tell if they are linearly dependent or linearly independent?

$$v_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}, v_4 = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}.$$

(3b) Are  $v_1, v_2$  and  $v_3$  linearly dependent or linearly independent?

(3b) Are  $v_1, v_2$  and  $v_4$  linearly dependent or linearly independent?

(4)  $A$  is a 9 by 13 matrix;  $A$  reduces to a matrix in RREF with exactly 2 zero rows. What is the rank of  $A$ ? What is the nullity of  $A$ ?

(5)  $B = \begin{bmatrix} 1 & 4 & 0 \\ 4 & 14 & 4 \\ 0 & 4 & 0 \end{bmatrix}$  Find the inverse of  $B$ .

(6) A 3 by 3 matrix  $A$  reduces to the identity by the following row operations:

- 1) Add  $2R_1$  to  $R_2$
- 2) Add  $-3R_1$  to  $R_3$
- 3) Multiply  $R_3$  by  $1/2$
- 4) Add  $-R_3$  to  $R_2$
- 5) Add  $-3R_2$  to  $R_1$

(6a) What is  $A^{-1}$ ?

(6b) Express  $A^{-1}$  as a product of elementary matrices.

(6c) Express  $A$  as a product of elementary matrices.

(7)  $A = \begin{bmatrix} 1 & 2 & 3 & 1 & 6 & 5 \\ 1 & 2 & 3 & 2 & 9 & 7 \\ 2 & 4 & 6 & 1 & 9 & 8 \end{bmatrix}$

(7a) What is the rank of  $A$ ? What is the nullity of  $A$ ?

(7b) Are the rows of  $A$  linearly independent?

(7c) Are the columns of  $A$  linearly independent?

(7d) Find all solutions of  $Ax = 0$ .