**M308 Quiz #1** Jan 10, 2007, **Solutions**

**(1a)** For the following system, form the augemnted matrix.

**(1b)** Use EROS's to transform the system to reduced row echelon form (RREF).

**(1c)** Find all solutions of the system.

$$\begin{array}{rcrcrcrcr} x_1 & + & 3x_2 & - & 2x_3 & + & 4x_4 & = & 2 \\ -3x_1 & - & 9x_2 & + & 7x_3 & - & 8x_4 & = & -3 \\ 2x_1 & + & 6x_2 & - & x_3 & + & 20x_4 & = & 13 \end{array}$$

**Solution**

$$\left[\begin{array}{rrrr|r} 1 & 3 & -2 & 4 & 2 \\ -3 & -9 & 7 & -8 & -3 \\ 2 & 6 & -1 & 20 & 13 \end{array}\right] \rightsquigarrow \left[\begin{array}{rrrr|r} 1 & 3 & -2 & 4 & 2 \\ 0 & 0 & 1 & 4 & 3 \\ 2 & 6 & -1 & 20 & 13 \end{array}\right] \rightsquigarrow \left[\begin{array}{rrrr|r} 1 & 3 & -2 & 4 & 2 \\ 0 & 0 & 1 & 4 & 3 \\ 0 & 0 & 3 & 12 & 9 \end{array}\right] \rightsquigarrow$$

$$\left[\begin{array}{rrrr|r} 1 & 3 & -2 & 4 & 2 \\ 0 & 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightsquigarrow \left[\begin{array}{rrrr|r} 1 & 3 & 0 & 12 & 8 \\ 0 & 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Solutions are $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 8 - 12x_4 - 3x_2 \\ x_2 \\ 3 - 4x_3 \\ x_4 \end{bmatrix}$, with $x_4$ and $x_2$ arbitrary.

**(2)** $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $w = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ Find numbers $x_1$ and $x_2$ so that $w = x_1v_1 + x_2v_2$.

**Solution**

$$\left[\begin{array}{rr|r} 1 & 3 & 2 \\ 2 & 5 & 6 \end{array}\right] \rightsquigarrow \left[\begin{array}{rr|r} 1 & 3 & 2 \\ 0 & -1 & 2 \end{array}\right] \rightsquigarrow \left[\begin{array}{rr|r} 1 & 3 & 2 \\ 0 & 1 & -2 \end{array}\right] \rightsquigarrow \left[\begin{array}{rr|r} 1 & 0 & 8 \\ 0 & 1 & -2 \end{array}\right]; \quad x_1 = 8,\ x_2 = -2.$$

Check: $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot 8 + \begin{bmatrix} 3 \\ 5 \end{bmatrix} \cdot (-2) = \begin{bmatrix} 8 - 6 \\ 16 - 10 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$