M308 Test #1 Jan 31, 2007 **SOLUTIONS**

(1) $A = \begin{bmatrix} 1 & -4 \\ -3 & 14 \end{bmatrix}$ **(1a)** What is A^{-1}?

SOLUTION The symbol $\rightsquigarrow$ stands for "row-reduces to"

$$\left[\begin{array}{cc|cc} 1 & -4 & 1 & 0 \\ -3 & 14 & 0 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|cc} 1 & -4 & 1 & 0 \\ 0 & 2 & 3 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|cc} 1 & 0 & 7 & 2 \\ 0 & 2 & 3 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|cc} 1 & 0 & 7 & 2 \\ 0 & 1 & \frac{3}{2} & \frac{1}{2} \end{array}\right]$$

So $A^{-1} = \begin{bmatrix} 7 & 2 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$

(1b) A^{-1} can be written as a product of three elementary matrices: $A^{-1} = E_3E_2E_1$. What are E_1, E_2, and E_3?

SOLUTION

$$E_1 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad E_3 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}.$$

(1c) A can be written as a product of three elementary matrices: $A = F_1F_2F_3$. What are F_1, F_2, and F_3?

SOLUTION

$$F_1 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \quad F_2 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \quad F_3 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix};$$

(2) $A = \begin{bmatrix} 1 & 2 & 1 & 2 & 2 \\ 1 & 2 & 2 & 3 & 4 \\ 2 & 4 & -1 & 1 & -2 \end{bmatrix}$. $x = \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix}$. $y = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

(2a) Is x in the columnspace of A?

SOLUTION We ask if there is a solution $t = [t_1, t_2, t_3, t_4, t_5]$ to $At = x$.

$$[A|x] = \left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 2 & 4 \\ 1 & 2 & 2 & 3 & 4 & 5 \\ 2 & 4 & -1 & 1 & -2 & 5 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

The answer is YES, $x \in C(\mathrm{A})$, and we may take $t_5 = 0$, $t_4 = 0$, $t_3 = 1$, $t_2 = 0$, and $t_1 = 3$

Then $x = 3 \cdot c_1 + 1 \cdot c_3$, where c_1, c_2, c_3, c_4, c_5 are the columns of A.

(2a') Is y in the columnspace of A?

SOLUTION We ask if there is a solution $t = [t_1, t_2, t_3, t_4, t_5]$ to $At = y$. Then

$$[A|y] = \left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 2 & 4 \\ 1 & 2 & 2 & 3 & 4 & 5 \\ 2 & 4 & -1 & 1 & -2 & 6 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right].$$

The answer is NO, y is not in $Col(\mathrm{A})$, because the equations (for the t_i) are inconsistent.

(2b) $v = [3, 6, 2, 5, 4]$. Is v in the rowspace of A? If yes, exhibit v as a linear combination of the rows of A. If no, explain why v cannot be a linear combination of the rows.

SOLUTION

$$[A^T|v^T] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 2 & 2 & 4 & 6 \\ 1 & 2 & -1 & 2 \\ 2 & 3 & 1 & 5 \\ 2 & 4 & -2 & 4 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 5 & 3 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{YES, } v \text{ is in } Row(\mathrm{A})\text{: } v = 3 \cdot r_1 - 1 \cdot r_2$$

(2c) $w = [1, -1, 1, 1, -1]$. Is w^T in the nullspace of A? Explain your answer.

SOLUTION $A \cdot w^T = \begin{bmatrix} 1 - 2 + 1 + 2 - 2 \\ 1 - 2 + 2 + 3 - 4 \\ 2 - 4 - 1 + 1 + 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, So YES, w^T is in $Null(A)$.