M308I Test # 1a Oct 24, 2007 **SOLUTIONS:**

(1) $A = \begin{bmatrix} 1 & 2 & 1 & 2 & 2 \\ 1 & 2 & 2 & 3 & 4 \\ 2 & 4 & -1 & 1 & -2 \end{bmatrix}, \quad x = \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix}, \quad w = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}.$

(1a) (10 pts.) Is w in the nullspace of A? Explain your answer.

Solution: $A \cdot w = \begin{bmatrix} 1 & 2 & 1 & 2 & 2 \\ 1 & 2 & 2 & 3 & 4 \\ 2 & 4 & -1 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1-2+1+2-2 \\ 1-2+2+3-4 \\ 2-4-1+1+2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$

So the **ANSWER** is YES, w is in $Null(A)$, because $A \cdot w = 0$.

(1b) (15 pts.) Is x in the columnspace of A? Explain your answer.

Solution: We ask if there is a solution $t = [t_1, t_2, t_3, t_4, t_5]$ to $At = x$.

$$[A|x] = \left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 2 & 4 \\ 1 & 2 & 2 & 3 & 4 & 5 \\ 2 & 4 & -1 & 1 & -2 & 5 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

The answer is YES, $x \in C(\mathrm{A})$, and we may take $t_5 = 0$, $t_4 = 0$, $t_3 = 1$, $t_2 = 0$, and $t_1 = 3$

Then $x = 3 \cdot c_1 + 1 \cdot c_3$, where c_1, c_2, c_3, c_4, c_5 are the columns of A.

(1c) (15 pts) $v = [3, 6, 2, 5, 4]$. Is v in the rowspace of A? Explain your answer.

Solution. $$[A^T|v^T] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 2 & 2 & 4 & 6 \\ 1 & 2 & -1 & 2 \\ 2 & 3 & 1 & 5 \\ 2 & 4 & -2 & 4 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 5 & 3 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So the **Answer** is YES, v is in $Row(\mathrm{A})$: $v = 3 \cdot r_1 - 1 \cdot r_2$

(1d) (5 pts) What is the rank of A? **Answer:** rank = 2

(1e) (5 pts) What is the nullity of A? **Answer:** nullity = 5 -2 = 3

(2) (15 pts) $v = [1, 1, 2]$, $w = [2, 6, 5]$. Find a scalar c and a vector y with $w = cv + y$, and $y \perp v$.

Solution: $0 = v^T \cdot (w - cv) = 18 - c \cdot 6$, so $c = 3$

$$y = w - c \cdot v \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix} - 3 \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$$

(3) $A = \begin{bmatrix} 1 & -4 \\ -3 & 11 \end{bmatrix}$ **(3a)** (15 pts) What is A^{-1}?

Solution: $[A|I] = \left[\begin{array}{cc|cc} 1 & -4 & 1 & 0 \\ -3 & 11 & 0 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|cc} 1 & -4 & 1 & 0 \\ 0 & -1 & 3 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|cc} 1 & -4 & 1 & 0 \\ 0 & 1 & -3 & -1 \end{array}\right] \rightsquigarrow$

$\left[\begin{array}{cc|cc} 1 & 0 & -11 & -4 \\ 0 & 1 & -3 & -1 \end{array}\right]$, so $A^{-1} = \begin{bmatrix} -11 & -4 \\ -3 & -1 \end{bmatrix}$,

(3b) (10 pts) A^{-1} can be written as a product of three elementary matrices: $A^{-1} = E_3 E_2 E_1$. What are E_1, E_2, and E_3?

Answer: $E_1 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ $E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $E_3 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

(3c) (10 pts) A can be written as a product of three elementary matrices: $A = F_1 F_2 F_3$. What are F_1, F_2, and F_3?

Answer: $F_1 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ $F_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $F_3 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$