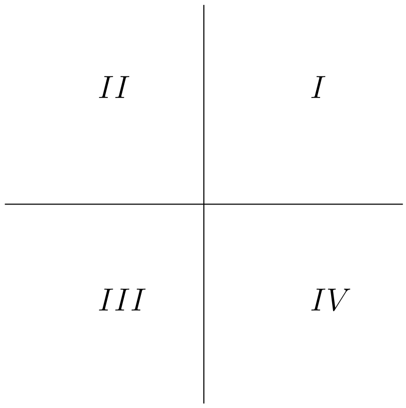
M309 Sample Problems on Systems of ODE #1 Jan 24, 2007 Name: ....

(1a) Let $A = \begin{bmatrix} 3 & 2 \\ -2 & -2 \end{bmatrix}$ Find the general solution to $x' = Ax$.

(1b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

(1c) The quadrants are numbered in the diagram below The trajectory of **(1b)** starts in quadrant I. In which quadrant (I, II, III or IV) will the trajectory be when $t = 1000$?



(2a) Let $A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}$ Find the general solution to $x' = Ax$.

(2b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$.

(2c) The four quadrants are numbered as in the diagram of **(1c)**. The trajectory of **(2b)** starts in quadrant I. What are the next four quadrants through which the trajectory passes?

(3a) Let $A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$ Find the general solution to $x' = Ax$.

(3b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$.

(4a) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$

(4b) Find a matrix T and a diagonal matrix D so that $T^{-1}AT = D$.

(5a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 6 & 1 \\ -3 & 2 \end{bmatrix}$.

(5b) Find an invertible matrix T and a diagonal matrix D such that $A = TDT^{-1}$.

(6a) Find the general solution to $\begin{array}{rcrcr} x_1' & = & 6x_1 & + & x_2 \\ x_2' & = & -3x_1 & + & 2x_2 \end{array}$

(6b) Find the solution to (2a) with $x_1(0) = 1$ and $x_2(0) = 3$.

(7a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 7 & 5 \\ -1 & 1 \end{bmatrix}$.

(7b) Find an invertible matrix T and a diagonal matrix D such that $A = TDT^{-1}$.

(8a) Find the general solution to $\begin{array}{rcrcr} x_1' & = & 7x_1 & + & 5x_2 \\ x_2' & = & -x_1 & + & x_2 \end{array}$

(8b) Find the solution to (2a) with $x_1(0) = 1$ and $x_2(0) = 3$.

(9) Find the general solution to the system $\begin{array}{l} x_1' = x_1 + x_2 + e^t \\ x_2' = 4x_1 + x_2 - 2e^t \end{array}$

(10) (Section 9.2 # 11:) $(0.5, 0.5)$ is a critical point of the system $\begin{cases} x' = -x + 2xy \\ y' = y - x^2 - y^2 \end{cases}$

For the critical point $(0.5, 0.5)$ find the corresponding approximate linear system and find the eigenvalues and eigenvectors. Classify the critical point $(0.5, 0.5)$ by type, and determine whether or not it is **AS** (asymptotically stable), **S** but not **AS** (stable but not asymptotically stable), or **US** (unstable).

(11a) Find the general solution to the system $\begin{aligned} x_1' &= x_1 + x_2 \\ x_2' &= 4x_1 + x_2 \end{aligned}$

(11b) Find the solution to the system $\begin{aligned} x_1' &= x_1 + x_2 \\ x_2' &= 4x_1 + x_2 \end{aligned}$ with $x_1(1) = 3$, $x_2(1) = 2$.

(12) Find all critical points of the system: $\begin{cases} x' = x(1.5 - 0.5x - y) \\ y' = y(0.75 - y - 0.125x) \end{cases}$

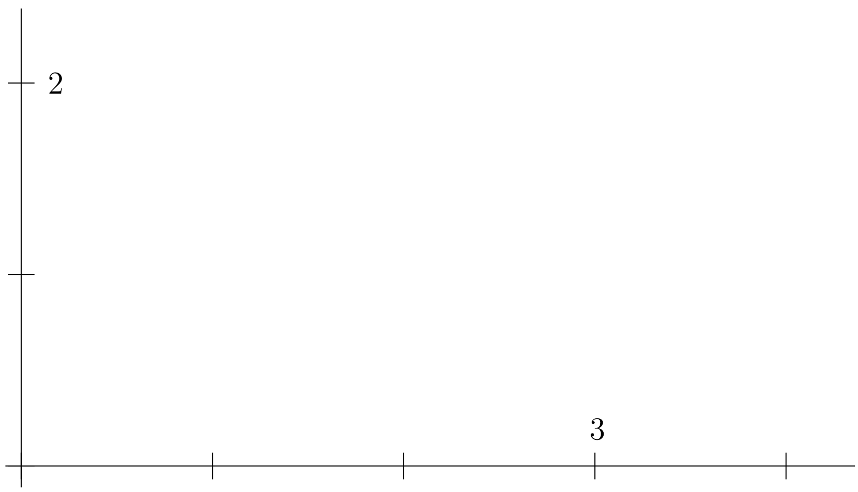
(13) Two populations $x(t)$ and $y(t)$ satisfy: $\begin{cases} x' = x(1 - 0.5x - 0.5y) \\ y' = y(-0.25 + 0.5x) \end{cases}$

The critical points are $(0,0)$, $(2,0)$ and $(0.5, 1.5)$.

(13a) For each critical point, find the corresponding approximate linear system.

(13b) Classify each critical point as to type and stability.

(13c) Plot enough of the direction field to determine the behavior of the solutions. Determine the limiting behavior of $(x(t), y(t))$ as $t \longrightarrow \infty$.



(14) The populations of two species satisfy: $x' = x(1 - 0.5x - 0.5y)$; $y' = y(-0.25 + 0.5x)$.

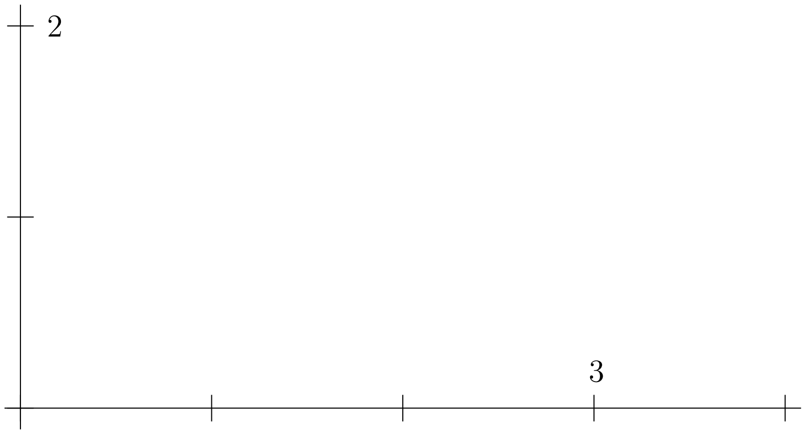
(14a) Find all critical points of the system.

(14b) For each critical point, find the corresponding approx. linear system.

(14c) For each critical point, find the eigenvalues, eigenvectors of the approx. linear system.

(14d) Classify each critical point as to type and stability.

(14e) Plot enough of the direction field to determine the behavior of the solutions. Determine the limiting behavior of $(x(t), y(t))$ as $t \longrightarrow \infty$, and interpret the results.



(15) The populations of predator, prey satisfy: $x' = x(1.125 - x - 0.5y)$; $y' = y(-1 + x)$.

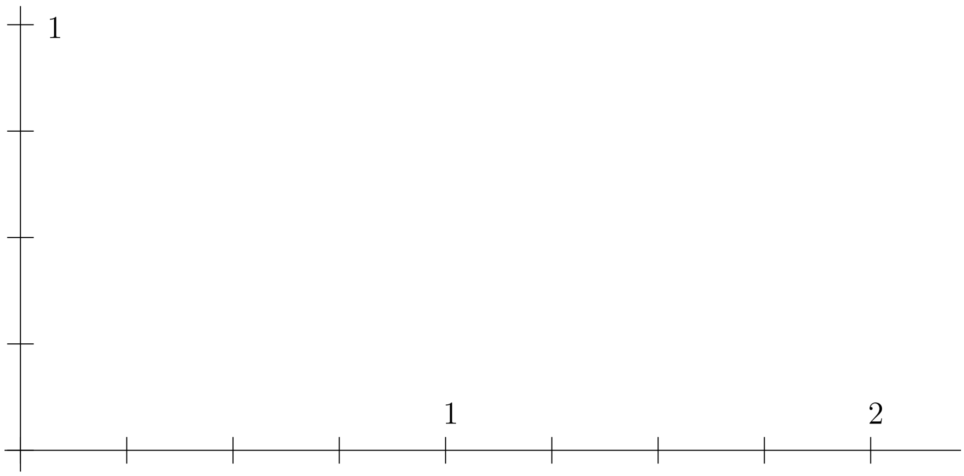
(15a) Find all critical points of the system.

(15b) For each critical point, find the corresponding approximating linear system.

(15c) For each critical point, find eigenvalues, eigenvectors of the approx. linear system.

(15d) Classify each critical point as to type and stability.

(15e) Plot enough of the direction field to determine the behavior of the solutions.



M309 Sample Final #1 Dec 6, 2000 Name: ....

(1a) Find the general solution to $x' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} x$

(1b) Classify the origin of the system in (3a) as node, saddle, center or spiral.

(1c) Is system in (3a) as stable or unstable.

(1d) Find the solution to $x' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} x$ with with $x_1(0) = 2$ and $x_2(0) = -4$.

(2) The populations of two species satisfy: $x' = x(1 - x + 0.5y)$; $y' = y(2.5 - 1.5y + 0.25x)$.

(2a) Find all critical points of the system.

(2b) For each critical point, find the corresponding approximate linear system.

(2c) For each critical point, find the eigenvalues, eigenvectors of the approximate linear system.

(2d) Classify each critical point as to type and stability.

(2e) Plot enough of the direction field to determine the behavior of the solutions. Determine the limiting behavior of $(x(t), y(t))$ as $t \longrightarrow \infty$, and interpret the results.

M309 Sample Final #2 Name: ....

(1a) Let $A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$. Find the general solution to $x' = Ax$.

(1b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

(1c) Describe the behavior of $x(t)$ as $t \longrightarrow \infty$. Sketch the trajectory.

(2) The populations of two species satisfy: $x' = x(1 - 0.5x - 0.5y)$; $\quad y' = y(-0.25 + 0.5x)$.

(2a) Find all critical points of the system.

(2b) For each critical point, find the corresponding approximate linear system.

(2c) For each critical point, find the eigenvalues, eigenvectors of the approximate linear system.

(2d) Classify each critical point as to type and stability.

(2e) Plot enough of the direction field to determine the behavior of the solutions. Determine the limiting behavior of $(x(t), y(t))$ as $t \longrightarrow \infty$, and interpret the results.